MAT511 homework, due Nov 4, 2009

(1) Suppose that A is a finite set with m elements, and B is a finite set with n elements.

 (a) Find the total number of functions from A to B if
  - $m = n$
  - $m > n$
  - $m < n$

 (b) Find the number of one-to-one functions from A to B if
  - $m = n$
  - $m > n$
  - $m < n$

(2) Give an example of functions $f : A \to B$ and $g : B \to C$, (be sure to specify domains and ranges) for which
 (a) g is onto C, but $g \circ f$ is not onto C.
 (b) $g \circ f$ is onto C, but f is not onto B.
 (c) g is one-to-one, but $g \circ f$ is not one-to-one.
 (d) $g \circ f$ is one-to-one but g is not one-to-one.

(3) Let $f : \mathbf{R} \to \mathbf{R}$ be given by $f(x) = x^2 + 1$. Find the following (remember that in this context $[a, b]$ is the set $\{x \in \mathbf{R} | a \le x \le b\}$).
 (a) $f([1, 2])$
 (b) $f([-1, 2])$
 (c) $f^{-1}([5, 10])$
 (d) $f^{-1}([-1, 5] \cup [17, 26])$

(4) Let $f : A \to B$, and $D \subseteq A$, $E \subseteq B$.
 (a) Prove that $D \subseteq f^{-1}(f(D))$. Also, Give an example where $D \ne f^{-1}(f(D))$.
 (b) Prove that $f(f^{-1}(E)) \subseteq E$. Also, Give an example where $f(f^{-1}(E)) \ne E$.

(5) (a) Suppose $y = f(x) = 3x - 47$. Write x as a function of y.
 (b) Suppose $y = f(x) = \frac{3x-2}{5x+7}$. What is the domain of f? Write $x = g(y)$ as a function of y. What is the domain of g? Check that the domain of g is the range of f, and vice-versa.