Problem 1. Given a circle of inversion centered at a point $O$, and a point $A$ (different from $O$), construct the image $A'$ of the point $A$ under the inversion, using a compass and straightedge. Justify your construction. For a hint, see the figure on page 1 of the lecture notes.

Problem 2. In class, we proved that an inversion will map a circle $c$ to a circle, provided that the original circle does not pass through the center $O$ of the inversion. (See Theorem 1.C in lecture notes.) We pictured the case where the circle $c$ does not intersect the circle of inversion, assuming in addition that $O$ lies outside $c$. Illustrate some of the remaining cases:

(a) the circle $c$ intersects the circle of inversion; $O$ lies outside $c$. Sketch the image of $c$ under inversion. Justify the features of your picture.

(b) the circle $c$ lies inside the circle of inversion; $O$ lies inside $c$. Again, the image of $c$ under inversion, explain and justify the prominent features of your picture.

For each case, prove that the image of $c$ under the inversion is a circle.

(Follow the line of proof give in class; explain how to adapt it to your cases).

Problem 3. Prove that the composition of two inversions with the same center is a homothety centered at the same center. What is the ratio of this homothety?

Problem 4. Prove that inversions preserve angles in the following case (not considered in class). Suppose $l$ is a line that passes through $O$ (the center of inversion), $m$ is a line that does not pass through $O$. Show that the angle (at the appropriate intersection point) between images of $l$ and $m$ under inversion is the same as the angle between $l$ and $m$.

Problem 5. Let $C$ be a circle of radius $R$, centered at $O$. Consider the inversion $\text{Inv}_{O,R}$ about this circle. Let $c$ be another circle such that $c$ intersects $C$ at two points, and at these intersection points the circles $c$ and $C$ (i.e. their tangent lines) are perpendicular. Show that $\text{Inv}_{O,R}$ maps $c$ to itself.