MAT 360 Geometry

Homework 11

due Thursday, May 9

Problem 1. Given a circle of inversion centered at a point O, and a point A (different from O), construct the image A' of the point A under the inversion, using a compass and straightedge. Justify your construction. For a hint, see the figure on page 1 of the lecture notes.

Problem 2. In class, we proved that an inversion will map a circle c to a circle, provided that the original circle does not pass through the center O of the inversion. (See Theorem 1.C in lecture notes.) We pictured the case where the circle c does not intersect the circle of inversion, assuming in addition that O lies outside c. Illustrate some of the remaining cases:

(a) the circle c intersects the circle of inversion; O lies outside c. Sketch the image of c under inversion. Justify the features of your picture.

(b) the circle c lies inside the circle of inversion; O lies inside c. Again, the image of c under inversion, explain and justify the prominent features of your picture.

For each case, prove that the image of c under the inversion is a circle. (Follow the line of proof give in class; explain how to adapt it to your cases).

Problem 3. Prove that the composition of two inversions with the same center is a homothety centered at the same center. What is the ration of this homothety?

Problem 4. Prove that inversions preserve angles in the following case (not considered in class). Suppose l is a line that passes through O (the center of inversion), m is a line that does not pass through O. Show that the angle (at the appropriate intersection point) between images of l and m under inversion is the same as the angle between l and m.

Problem 5. Let C be a circle of radius R, centered at O. Consider the inversion $Inv_{O,R}$ about this circle. Let c be another circle such that c intersects C at two points, and at these intersection points the circles c and C (i.e. their tangent lines) are perpendicular. Show that $Inv_{O,R}$ maps c to itself.