Question 1. This is a question about limits.

Fix $x_0 \in \mathbb{R}$, let $(a, b)$ be an interval containing $x_0$, and suppose that $f(x)$ is defined for all $x \in (a, b)$, $x \neq x_0$.

(a) Prove that if $f(x) \geq 0$ for all $x \in (a, b)$, $x \neq x_0$, then $\lim_{x \to x_0} f(x) \geq 0$.

(b) Prove that if $f(x) \leq 0$ for all $x \in (a, b)$, $x \neq x_0$, then $\lim_{x \to x_0} f(x) \leq 0$.

(c) Give an example of a function $f_1$ such that $f_1(x) > 0$ for all $x \in (a, b)$, $x \neq x_0$ but $\lim_{x \to x_0} f_1(x) = 0$, and also of function $f_2$ such that $f_2(x) < 0$ for all $x \in (a, b)$, $x \neq x_0$ but $\lim_{x \to x_0} f_2(x) = 0$.

Question 2. In class, we showed that if the function $f$ increases on $(a, b)$, then $f'(x) \geq 0$ for all $x \in (a, b)$.

Use the same strategy to show that if $f$ decreases on $(a, b)$ then $f'(x) \leq 0$ for all $x \in (a, b)$.

Give an example of a function $f$ that strictly decreases on $(a, b)$ but $f'(x_0) = 0$ for some $x_0 \in (a, b)$.

In this question, all functions we consider are assumed to be differentiable on their domain.

Please also do questions 28.2, 28.4, 28.8, 28.14.