MAT 319
Homework 11
due Wednesday, April 27

Question 1. This is a question about limits.
Fix $x_0 \in \mathbb{R}$, let (a, b) be an interval containing x_0, and suppose that $f(x)$ is defined for all $x \in (a, b)$, $x \neq x_0$.
(a) Prove that if $f(x) \geq 0$ for all $x \in (a, b)$, $x \neq x_0$, then $\lim_{x \to x_0} f(x) \geq 0$.
(b) Prove that if $f(x) \leq 0$ for all $x \in (a, b)$, $x \neq x_0$, then $\lim_{x \to x_0} f(x) \leq 0$.
(c) Give an example of a function f_1 such that $f_1(x) > 0$ for all $x \in (a, b)$, $x \neq x_0$ but $\lim_{x \to x_0} f_1(x) = 0$, and also of function f_2 such that $f_2(x) < 0$ for all $x \in (a, b)$, $x \neq x_0$ but $\lim_{x \to x_0} f_2(x) = 0$.

Question 2. In class, we showed that if the function f increases on (a, b), then $f'(x) \geq 0$ for all $x \in (a, b)$.
Use the same strategy to show that if f decreases on (a, b) then $f'(x) \leq 0$ for all $x \in (a, b)$.
Give an example of a function f that *strictly* decreases on (a, b) but $f'(x_0) = 0$ for some $x_0 \in (a, b)$.
In this question, all functions we consider are assumed to be differentiable on their domain.

Please also do questions **28.2, 28.4, 28.8, 28.14**.