Name: ____________________________ Core Competency Exam A

You do not need to show work. Answer on the line.  

1. Problem 1 The equation |x - 3| = 2 has: (1) no solutions, (2) a unique solution, (4) one positive and one negative solution, or (3) two positive solutions, (5) two negative solutions.

2. Problem 2 The set of all real numbers where |2x - 6| ≤ 4 is:
   (1) (1, 5), (5) (2, 10), (3) (−∞, 1) U (5, ∞), (4) (−∞, 1) U [5, ∞), or (5) [2/2, 10/2].

3. Problem 3 The reflection through the y-axis of the graph of y = f(x) is the graph of:
   (1) y = −f(x), (2) y = f(−x), or (3) y = −f(−x).

4. Problem 4 For the function f(x) = 1/(x + 1), x ≥ 0, the value f(0) equals:
   (1) 1, (2) 0, (3) 1/2, (4) 2, or (5) undefined. 

5. Problem 5 For the functions f(x) = 1/x + 1, x ≠ 0, and g(u) = 1/(u + 1), u ≠ −1, the composite function f(g(x)), x ≠ −1, equals f(g(x)) = 1/(g(x) + 1) = 1/(1/x + 1) + 1 = (1/x + 1) - 1 = x + 2

6. Problem 6 The equation of the line with slope 2 containing the point (x, y) = (1, 3) is:
   (1) y = 2x + 1, (2) y = 2x + 3, or (4) y = 3x - 3(1 - x).

7. Problem 7 The line containing the two points (x, y) = (1, 1) and (2, −1) has equation 
   m = ∫ y - 1 = 2 - 1 = -2. (y - 1) = -2(x - 1) = -2x + 2, (y - 3) = -2x + 3

8. Problem 8 The perpendicular line to y = x + 2, containing the point (1, −1) has equation
   (1) y = 2x - 3, (2) y = 3x, (3) y = −1/2x + (−1/2), or (4) y = (1/2)x - 3/2.

9. Problem 9 The solutions of the quadratic equation 2x^2 + 4x = 0 are (1) x = 0 and x = −4, (2) x = 0 and x = −2, (3) x = −2 and x = −4, or (4) undefined. 2x(x + 2) = 0, x = {}.

10. Problem 10 The parabola with equation y = x^2 - 4x + 3 satisfies y > 0 for x in
    (1) (1, 3), (2) [-1, 1], (3) (−∞, 1) U (3, ∞), (4) (−∞, 1) U [3, ∞).
Mastery Exam. Show all Work.

Name: ____________________________  Problem 1: ____________/35

**Mastery Problem 1** (35 points) For all parts of this problem, \( f(x) \) equals \( \sqrt{9 - 2x} \). Show all work.

(a) (5 points) Find the domain of \( f \), i.e., the maximal set of real numbers for which the expression is defined as a real number. Express your answer using interval notation.

Domain of \( f \) defined for \( t \geq 0 \). Thus \( \sqrt{9 - 2x} \) defined for \( 9 - 2x \geq 0 \)

\[
\begin{align*}
9 - 2x & \geq 0 \\
2x & \leq 9 \\
\frac{9}{2} & \geq x
\end{align*}
\]

Domain equals \((-\infty, \frac{9}{2}]\).

(b) (10 points) Find the unique real number \( c \) such that \( f(c) \) equals 4.

\[
\begin{align*}
f(c) &= \sqrt{9 - 2c} = 4 \\
(\sqrt{9 - 2c})^2 &= (4)^2 \\
9 - 2c &= 16 \\
-2c &= 16 - 9 = 7 \\
c &= \frac{7}{2} = \frac{-7}{2}
\end{align*}
\]
(c) (5 points) Find the range of $f$. Express your answer in interval notation.

Method I

$y = \sqrt{9 - 2x}$

Range = $[0, \infty)$.

Method II

$y = \sqrt{9 - 2x}$ has solution if and only if $y \geq 0$,

namely $x = \frac{9 - y^2}{2} \leq \frac{9}{2}$

Range = $[0, \infty)$.

(d) (10 points) Find a formula for the inverse function $f^{-1}(y)$.

\[
\begin{align*}
    y &= \sqrt{9 - 2x} \\
    (y)^2 &= (\sqrt{9 - 2x})^2 \\
    y^2 &= 9 - 2x \\
    y^2 - 9 &= -2x \\
    \frac{y^2 - 9}{-2} &= x \\
    f^{-1}(y) &= -\frac{1}{2}y^2 + \frac{9}{2}
\end{align*}
\]

(e) (5 points) Write the domain and range of the inverse function $f^{-1}$. Write your answer in interval notation, and make clear which is the domain and which is the range.

\[
\begin{align*}
    f(x) &\quad f^{-1}(y) \\
    \text{Domain: } (-\infty, \frac{9}{2}] &\quad \text{Domain: } [0, \infty) \\
    \text{Range: } [0, \infty) &\quad \text{Range: } (-\infty, \frac{9}{2}]
\end{align*}
\]
Mastery Problem 2 (35 points) Beginning with the equation whose graph is the upper semicircle of radius 1 centered at the origin,

\[ g(x) = \sqrt{1 - x^2}, \]

**FIRST** scale horizontally (left-right) by 2 and scale vertically (up-down) by 4, and **NEXT** translate horizontally by 2 and translate vertically by $-1$. Find the equation $h(x)$ whose graph is the transformed graph. Please express your answer in the form

\[ h(x) = e + \sqrt{ax^2 + bx + c}, \]

for real numbers $a$, $b$, $c$ and $e$. Draw a rough sketch indicating the images on the new graph of the special points $(-1,0)$, $(0,1)$ and $(1,0)$ of the original graph. Show all work.

**General formula for vertical scale by $t$, horizontal scale by $s$ followed by vertical shift by $k$, horizontal shift by $h$.**

\[ \frac{y-k}{t} = f\left(\frac{x-h}{s}\right). \]

\[
\begin{align*}
\frac{y-(-1)}{4} &= g\left(\frac{x-2}{2}\right) = \sqrt{1 - \left(\frac{x-2}{2}\right)^2} = \sqrt{1 - \frac{(x-2)^2}{4}} = \sqrt{\frac{4 - (x^2 - 4x + 4)}{4}} = \sqrt{\frac{-x^2 + 4x}{4}} \\
y + 1 &= \sqrt{-x^2 + 4x} \\
y + 1 &= \sqrt{-\frac{x^2 + 4x}{4}} \\
y + 1 &= \sqrt{-\frac{x^2 + 4x}{4}} \\
y + 1 &= \sqrt{-\frac{x^2 + 4x}{4}} \\
\end{align*}
\]

\[ y = -1 + \sqrt{-\frac{4x^2 + 16}{4}} \]
Mastery Problem 3 (30 points) For the parabola with equation

\[ F(x) = -2x^2 - 4x - 3, \]

give a rough sketch of the parabola carefully labelling the vertex as well as all intersection points (if any) with both the x-axis and the y-axis. Show all work.

\[ \text{Vertex} \quad -2x^2 - 4x - 3 = -2(x^2 + 2x) - 3 = -2((x^2 + 2x + 1) - 1) - 3 = -2(x + 1)^2 + 2 - 3 = -2(x + 1)^2 - 1 \]

Since \( F(x) = -2(x + 1)^2 - 1 \), vertex is \((x, y) = (-1, -1)\).

There are no zeroes. \(-2(x + 1)^2 - 1 = 0\), \(-2(x + 1)^2 = 1\), \((x + 1)^2 = -\frac{1}{2}\) → Impossible, \(-\frac{1}{2}\) not nonnegative.

Y-intercept: \( F(0) = -2(0)^2 - 4(0) - 3 = 0 + 0 - 3 = -3 \)