Name: ______________________________ Core Competency Exam A

You do not need to show work. Answer on the line. $x = \begin{cases} 5, \text{ or} \\ 1 \end{cases}$

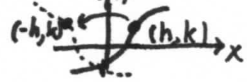
3. **Problem 1** The equation $|x-3| = 2$ has: (1) no solutions, (2) a unique solution, (4) one positive and one negative solution, or (3) two positive solutions, (5) two negative solutions.

2. **Problem 2** The set of all real numbers where $|2x-6| \leq 4$ is: $-4 \le 2x-6 \le +4$, $2 \le 2x \le 10$, $1 \le x \le 5$.

(1) $(1,5)$, (2) $[1,5]$, (3) $(-\infty,1) \cup (5,\infty)$, (4) $(-\infty,1] \cup [5,\infty)$, or (5) $(2/2, 10/2]$.

2. **Problem 3** The reflection through the y-axis of the graph of $y = f(x)$ is the graph of:

(1) $y = -f(x)$, (2) $y = f(-x)$, or (3) $y = -f(-x)$.



3. **Problem 4** For the function $f(x) = 1/(x+1)$, $x \geq 0$, the value $f(f(0))$ equals:

(1) 1, (2) 0, (3) 1/2, (4) 2, or (5) undefined. $f(0) = \frac{1}{0+1} = 1$
$f(1) = \frac{1}{1+1} = \frac{1}{2}$

3. **Problem 5** For the functions $f(x) = \frac{1}{x} + 1$, $x \neq 0$, and $g(u) = \frac{1}{u+1}$, $u \neq -1$, the composite function $f(g(x))$, $x \neq -1$, equals $f(g(x)) = \frac{1}{g(x)} + 1 = \frac{1}{\frac{1}{(x+1)}} + 1 = (x+1) + 1 = x+2$

(1) $\dfrac{x}{2x+1}$, (2) $\dfrac{1}{(1/x)+2}$, (3) $x+2$, (4) $\dfrac{1}{1/(x+1)} - 1$.

$y-3 = 2(x-1) = 2x-2$,

1. **Problem 6** The equation of the line with slope 2 containing the point $(x,y) = (1,3)$ is: $\underline{y = 2x+1}$

(1) $y = 2x+1$, (2) $y - 1 = 2(x-3)$, (3) $y = 2x+3$, or (4) $y - 3 = 3(x-1)$.

4. **Problem 7** The line containing the two points $(x,y) = (1,1)$ and $(x,y) = (2,-1)$ has equation

$m = \frac{-1-1}{2-1} = -2$. $(y-1) = -2(x-1) = -2x+2$,
$\underline{y = -2x+3}$

(1) $y - 1 = \dfrac{2-1}{-1-1}(x-1)$, (2) $y - 1 = \dfrac{-1-1}{2-1}(x-2)$, (3) $y = 1x+1$, or (4) $y = -2x+3$.

$m = 2$, $m' = -\frac{1}{m} = -\frac{1}{2}$. $(y-(-1)) = -\frac{1}{2}(x-1) = -\frac{1}{2}x + \frac{1}{2}$, $\underline{y = -\frac{1}{2}x - \frac{1}{2}}$

3. **Problem 8** The perpendicular line to $y = 2x + 2$, containing the point $(1,-1)$ has equation

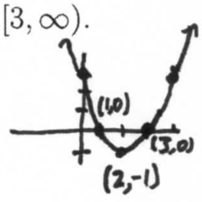
(1) $y = 2x-3$, (2) $y = (-1/2)x - 1$, (3) $y = (-1/2)x + (-1/2)$, or (4) $y = (1/2)x - 3/2$.

2. **Problem 9** The solutions of the quadratic equation $2x^2 + 4x = 0$ are (1) $x = 0$ and $x = -4$, (2) $x = 0$ and $x = -2$, (3) $x = -2$ and $x = -4$, or (4) undefined. $2x(x+2) = 0$, $x = \begin{cases} 0, \text{ or} \\ -2 \end{cases}$

3. **Problem 10** The parabola with equation $y = x^2 - 4x + 3$ satisfies $y > 0$ for x in

(1) $(1,3)$, (2) $[1,3]$, (3) $(-\infty,1) \cup (3,\infty)$, (4) $(-\infty,1] \cup [3,\infty)$.

$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$
$x = \begin{cases} 1 \\ 3 \end{cases}$



Mastery Exam. Show all Work.

Name: ______________________________ **Problem 1:** ____________ /35

Mastery Problem 1(35 points) For all parts of this problem, $f(x)$ equals $\sqrt{9-2x}$. Show all work.

(a)(5 points) Find the domain of f, i.e., the maximal set of real numbers for which the expression is defined as a real number. Express your answer using interval notation.

Domain. $\sqrt{t}$ defined for $t \geq 0$. Thus $\sqrt{9-2x}$ defined for $9-2x \geq 0$

$9 \geq 2x$

$\frac{9}{2} \geq x$

Domain equals $\boxed{(-\infty, \frac{9}{2}].}$

(b)(10 points) Find the unique real number c such that $f(c)$ equals 4.

$f(c) = \sqrt{9-2c} = 4$

$(\sqrt{9-2c})^2 = (4)^2$

$9-2c = 16$

$-2c = 16-9 = 7$

$c = \frac{7}{-2} = \frac{-7}{2}$

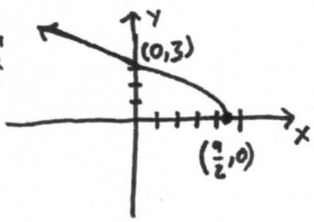
$\boxed{c = \frac{-7}{2}}$

Name: ______________________________ **Problem 1 continued.**

(c)(5 points) Find the range of f. Express your answer in interval notation.

Method I

$y=\sqrt{9-2x}$



Range $= [0,\infty)$.

Method II

$y=\sqrt{9-2x}$ has solution if & only if $y \geq 0$, namely $x = \frac{9-y^2}{2} \leq \frac{9}{2}$. ✓

Range $= \boxed{[0,\infty).}$

(d)(10 points) Find a formula for the inverse function $f^{-1}(y)$.

$$y = \sqrt{9-2x}$$
$$(y)^2 = (\sqrt{9-2x})^2$$
$$y^2 = 9-2x$$
$$y^2 - 9 = -2x$$
$$\frac{y^2-9}{-2} = x$$

$$\boxed{f^{-1}(y) = -\frac{1}{2}y^2 + \frac{9}{2}}$$

(e)(5 points) Write the domain and range of the inverse function f^{-1}. Write your answer in interval notation, and make clear which is the domain and which is the range.

$f(x)$

Domain: $(-\infty, \frac{9}{2}]$

Range: $[0,\infty)$

$f^{-1}(y)$

Domain: $[0,\infty)$

Range: $(-\infty, \frac{9}{2}]$

Name: ______________________________ **Problem 2:** ____________ /35

Mastery Problem 2(35 points) Beginning with the equation whose graph is the upper semicircle of radius 1 centered at the origin,

$$g(x) = \sqrt{1 - x^2},$$

FIRST scale horizontally (left-right) by 2 and scale vertically (up-down) by 4, and **NEXT** translate horizontally by 2 and translate vertically by -1. Find the equation $h(x)$ whose graph is the transformed graph. Please express your answer in the form

$$h(x) = e + \sqrt{ax^2 + bx + c},$$

for real numbers a, b, c and e. Draw a rough sketch indicating the images on the new graph of the special points $(-1, 0)$, $(0, 1)$ and $(1, 0)$ of the original graph. Show all work.

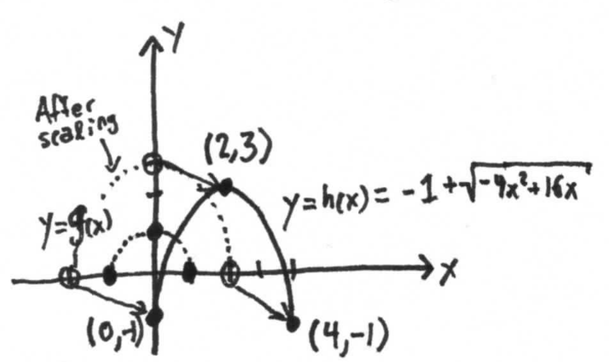
General formula for vertical scale by t, horizontal scale by s followed by vertical shift by k, horizontal shift by h.

$$\frac{y-k}{t} = g\left(\frac{x-h}{s}\right).$$

$$\frac{y-(-1)}{4} = g\left(\frac{x-2}{2}\right) = \sqrt{1-\left(\frac{x-2}{2}\right)^2} = \sqrt{1-\frac{(x^2-4x+4)}{4}} = \sqrt{\frac{4}{4}-\frac{(x^2-4x+4)}{4}} = \sqrt{\frac{-x^2+4x}{4}}$$

$$\frac{y+1}{4} = \sqrt{\frac{-x^2+4x}{4}}, \quad y+1 = 4\sqrt{\frac{-x^2+4x}{4}} = \sqrt{16}\sqrt{\frac{-x^2+4x}{4}} = \sqrt{\frac{16(-x^2+4x)}{4}} = \sqrt{4(-x^2+4x)}$$

$$y+1 = \sqrt{-4x^2+16x}. \quad \boxed{y = -1 + \sqrt{-4x^2+16x}}$$



Name: ______________________________ **Problem 3:** ______________ /30

Mastery Problem 3(30 points) For the parabola with equation

$$F(x) = -2x^2 - 4x - 3,$$

give a rough sketch of the parabola carefully labelling the vertex as well as all intersection points (if any) with both the x-axis and the y-axis. Show all work.

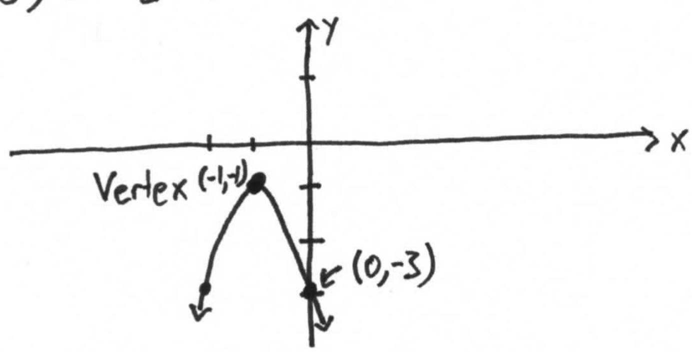
Vertex
$-2x^2-4x-3 = -2(x^2+2x)-3 = -2((x^2+2x+1)-1)-3 = -2(x+1)^2+2-3$
$= -2(x+1)^2-1$

Since $F(x) = -2(x+1)^2-1$, vertex is $(x,y) = (-1,-1)$.

Zeroes. $-2(x+1)^2-1=0$, $-2(x+1)^2 = 1$, $(x+1)^2 = -\frac{1}{2}$ → Impossible, $-\frac{1}{2}$ not nonnegative.

There are no zeroes.

y-intercept. $F(0) = -2(0)^2-4(0)-3 = 0+0-3 = -3$



$y=-2x^2-4x-3$