MAT 126 Summer II Midterm

Last Name: ______________ First Name: ______________ Student ID: ______________

Problem	1	2	3	4	5	Total	Bonus
Points	30	30	20	10	10	100	15
Scores							

This midterm has five problems, weighted as shown. Please show your work – full credit may not be given if only the answers appear. **No calculators or books will be allowed on this test.** When calculating indefinite integrals, the answers should be in explicit forms, unless otherwise stated.

1. Evaluate each of the following definite integrals.

(a) $\int_0^1 (\sqrt{x} + x^2)\,dx$

$x^{\frac{1}{2}} + x^2$

$\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{3}x^3 \Big|_0^1 \leftarrow$ 7

$\frac{2}{3} + \frac{1}{3} = 1 \leftarrow$ 3

(b) $\int_0^{\frac{\pi}{4}} \sin(3x)\,dx$

$u = 3x$ $\quad u \in [0, \frac{3\pi}{4}]$

3 $du = 3dx$

3 $\int_0^{\frac{3\pi}{4}} \sin u \cdot \frac{du}{3} = \frac{1}{3}(-\cos u)\Big|_0^{\frac{3\pi}{4}}$ 3

$= \frac{1}{3}(-\cos\frac{3\pi}{4}) + \frac{1}{3}\cos 0$

$= \frac{1}{3}\cdot\frac{\sqrt{2}}{2} + \frac{1}{3}$ 1

(c) $\int_0^2 t \ln t \, dt$

③ let $u = \ln t$; $du = \frac{1}{t} dt$ ②

$dv = t\,dt$; $v = \frac{1}{2}t^2$

$$\int u\,dv = uv - \int v\,du$$

$$= \frac{1}{2}t^2 \ln t \Big|_0^2 - \int_0^2 \frac{1}{2}t^2 \cdot \frac{1}{t}\,dt$$

$$= \frac{1}{2}t^2 \ln t \Big|_0^2 - \frac{1}{2}\int_0^2 t\,dt$$

④

$$= \left(\frac{1}{2}t^2 \ln t - \frac{1}{4}t^2\right)\Big|_0^2$$

$$= 2\ln 2 - 1 - \frac{1}{2}\lim_{t\to 0} \frac{\ln t}{t^{-2}}$$

$$= 2\ln 2 - 1 - \frac{1}{2}\lim_{t\to 0} \frac{\frac{1}{t}}{(-2)t^{-3}}$$

①

$$= 2\ln 2 - 1 + \lim_{t\to 0} t^2$$

$$= 2\ln 2 - 1$$

2. Calculate each of the following indefinite integrals.

(a) $\int x^5 \sin(x^3)\, dx$

2 $\rightarrow$ ~~[illegible]~~ let $w = x^3$ $\quad dw = 3x^2 dx$

5

3 $\rightarrow \int x^5 \sin(x^3)\, dx = \int w \cdot \sin w \frac{dw}{3}$

$= \frac{1}{3}\int w \sin w \, dw$

2 $\rightarrow$ let $\left.\begin{array}{l} u = w \\ dv = \sin w \, dw \end{array}\right\} \Rightarrow \begin{array}{l} du = dw \\ v = -\cos w. \end{array}$

3 $\rightarrow$ Int. by Parts ; $\frac{1}{3}\int w \sin w \, dw = \frac{1}{3}\left(w \cdot (-\cos w) + \int \cos w \, dw\right)$

5

$= \frac{1}{3}(-w \cdot \cos w + \sin w)$

$= \frac{1}{3}(-x^3 \cos(x^3) + \sin x^3)$

(b) $\int \frac{x-6}{x^2-6x+8}\, dx$

1 $\quad x^2 - 6x + 8 = (x-2)(x-4)$

want: $\frac{x-6}{x^2-6x+8} = \frac{x-6}{(x-2)(x-4)} = \frac{A}{x-2} + \frac{B}{x-4}$

3 $\quad \Rightarrow \quad x-6 = A(x-4) + B(x-2)$

3

let $x = 2 \Rightarrow 2-6 = A(2-4) \Rightarrow A = 2$

let $x = 4 \Rightarrow 4-6 = B(4-2) \Rightarrow B = -1$

3 then $\int \frac{x-6}{(x-2)(x-4)}\, dx = 2\int \frac{1}{x-2}\, dx - \int \frac{1}{x-4}\, dx$

$= 2\ln|x-2| - \ln|x-4| + C.$

(c) $\int \cos^2 x - \sin^2 x \, dx$

$\cos^2 x - \sin^2 x = \cos(2x)$ 4

$\int \cos(2x)\,dx$: let $u = 2x$
$du = 2dx$ 3

$\int \cos u \frac{du}{2} = \frac{1}{2}\sin u$

$= \frac{1}{2}\sin(2x) + C$ 3

3. Calculate each of the following indefinite integrals.

(a) $\int \tan^4 x \sec^6 x \, dx$ $= \int \tan^4 x \sec^4 x \, \underline{\sec^2 x \, dx}$

3 let $u = \tan x$ then $\sec^4 x = (\tan^2 x + 1)^2$

$du = \sec^2 x \, dx$ $= (u^2+1)^2$

3 then $\int u^4 (u^2+1)^2 \, du$

$= \int u^4 (u^4 + 2u^2 + 1) \, du$

2 $= \int (u^8 + 2u^6 + u^4) \, du$

1 $= \frac{1}{9} u^9 + \frac{2}{7} u^7 + \frac{1}{5} u^5 + C$

1 $= \frac{1}{9} \tan^9 x + \frac{2}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C.$

(b) $\int \frac{1}{t^3 \sqrt{t^2 - 4}} \, dt$

let ~~t = 2 sin~~ $t = 2\sec\theta$

3 $dt = 2\tan\theta \sec\theta \, d\theta$

$\sqrt{t^2 - 4} = 2\tan\theta$

$\int \frac{1}{t^3 \sqrt{t^2-4}} \, dt = \int \frac{1}{8\sec^{\cancel{2}3}\theta \cdot \cancel{2\tan\theta}} \cdot \cancel{2\tan\theta}\,\cancel{\sec\theta} \, d\theta$

$= \frac{1}{8} \int \frac{1}{\sec^2\theta} \, d\theta$

3 $= \frac{1}{8} \int \cos^2\theta \, d\theta$

2 $= \frac{1}{8} \int \frac{1 + \cos(2\theta)}{2} \, d\theta$

1 5 $= \frac{1}{16} \left(\theta + \frac{1}{2} \sin(2\theta)\right) + C.$

1 $= \frac{1}{16} \operatorname{arcsec}\left(\frac{t}{2}\right) + \frac{1}{32} \sin\left(2 \operatorname{arcsec}\left(\frac{t}{2}\right)\right) + C.$

4. Express the following limit as a definite integral. Do not evaluate the definite integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{4}{n}(3-\frac{2i}{n})$$

let $\Delta x = \frac{4}{n} \Rightarrow b-a=4$

$\Rightarrow a=0 \quad b=4$

② ↓

then $x_i = a+\frac{b-a}{n}\cdot i$

$= \frac{4i}{n} \qquad \Rightarrow \frac{x_i}{2} = \frac{2i}{n}$

④ ↓

$f(x_i) = 3-\frac{2i}{n} = 3-\frac{x_i}{2}$

③ ↓

$\Rightarrow f(x) = 3-\frac{x}{2}$

① ↓ then $\lim_{n\to\infty} \frac{4}{n}(3-\frac{2i}{n}) = \int_0^4 (3-\frac{x}{2})\,dx$

5. Find the derivative of the following function.

$$f(x) = \int_{e^x}^{0} 3\sin^2 t \, dt$$

$f(x) = \int_{e^x}^{0} 3\sin^2 t\,dt = -\int_{0}^{e^x} 3\sin^2 t\ dt$ 2

$u = e^x$

$f(x) = -\int_0^u 3\sin^2 t\,dt$

$\frac{df}{dx} = \frac{df}{du}\Big|_{u=u(x)} \cdot \frac{du}{dx}$

$= -3\sin^2(e^x) \cdot e^x$ 8

6. (Bonus) Evaluate the following indefinite integral.

$$\int \sin(\ln x)\, dx$$

let $y = \ln x$

3 then $x = e^y \Rightarrow dx = e^y dy$

$\int \sin(\ln x)\, dx$

2 $= \int \sin y \cdot (e^y dy)$

let $u = e^y \qquad\Rightarrow\qquad du = e^y dy$

$dv = \sin y\, dy \qquad\qquad v = -\cos y$

3 Int by Parts: $-e^y \cos y + \underline{\int \cos y \cdot e^y dy}$

For $\int \cos y\, e^y dy$: let $u = e^y \Rightarrow du = e^y dy$

$dv = \cos y\, dy \qquad v = \sin y$

3 Int by Parts: $\int \cos y\, e^y dy = e^y \sin y - \int \sin y\, e^y dy$

$\Rightarrow \int \sin y\, e^y dy = -e^y \cos y + e^y \sin y - \int \sin y\, e^y dy$

$\Rightarrow 2\int \sin y\, e^y dy = -e^y \cos y + e^y \sin y$

3 $\Rightarrow \int \sin y\, e^y dy = e^y(-\cos y + \sin y) + C$

$y = \ln x$: $e^{\ln x}(-\cos(\ln x) + \sin(\ln x)) + C$

1 $= x \cdot (-\cos(\ln x) + \sin(\ln x)) + C.$