
James Waterman Homework 0, Summing digits of π , MAT 331, Fall 2023

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Part 1: What is the sum of the first 10,000 digits of π ?

We use `vpa` to compute $N=10000$ digits of π and convert this to a character string `y` with `char`. We then let `x` hold the numerical value of the k th digit, being careful to omit the second character, which is the decimal point. Then `t` is defined as the sum of all the digits.

```
N=10000;  
y=char(vpa(pi,N));  
x(1)=str2num(y(1));  
y=char(vpa(pi,N));  
for k=2:N  
    x(k)=str2num(y(k+1));  
end  
t=sum(x)
```

`t =`

`44890`

This creates a single character string `y` that is 10,001 characters long (there is an extra character for the decimal point). We then convert this to a string of integers `x`, remembering to skip the decimal place. The answer to the first part is `t= 44890`.

Part 2: If the digits of π are uniformly random in $\{0, \dots, 9\}$, what would we expect the sum to be? How far apart are the actual and expected sums?

If the digits were uniformly random in $\{0, 1, \dots, 9\}$ then the average size of a digit would be $a = (0+1+\dots+9)/10 = 4.5$, and the sum of 10,000 such digits would be 45,000. The difference between this and the actual sum is $45000 - 44890 = 110$.

Part 3: For $1 \leq k \leq N$, plot the difference between the expected and the actual sum of the first k digits of π . Do you see any pattern?

To compute the sum of the first k digits of π we can either use a loop as follows,

```
c(1)=x(1);  
for k=2:N  
    c(k)=c(k-1)+x(k);  
end
```

or a built-in MATLAB command that does the same thing:

```
d=cumsum(x);
```

We can check that these both give the same list of numbers, by subtracting them and taking the maximum of the absolute value:

```
error = max(abs(c-d))
```

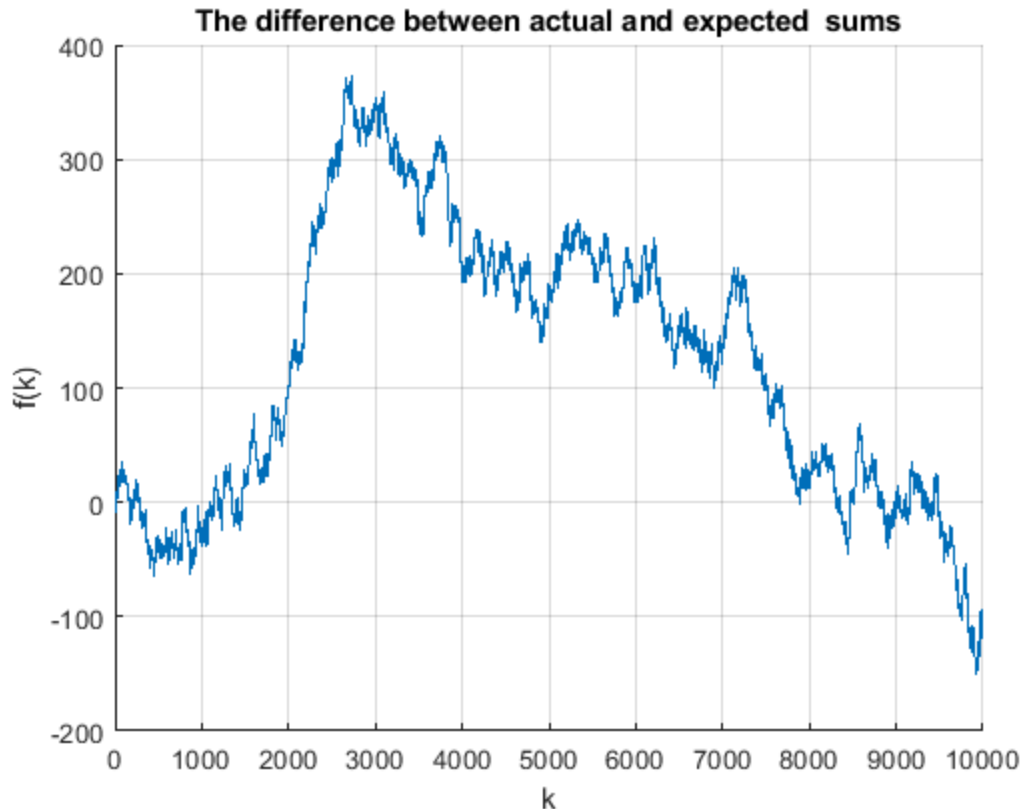
```
error =
```

```
0
```

To plot the difference between c and $4.5 k$ we use:

```
figure;           % creates an empty figure window  
hold on;          % tells MATLAB keep all the following features  
grid on;          % creates a grid  
title('The difference between actual and expected sums')  
f=c-4.5*[1:N];    % defines difference between actual sum and expected sum  
plot(f);          % plots f, default color is blue  
xlabel('k');       % puts label on horizontal axis  
ylabel('f(k)');    % puts label on vertical axis
```

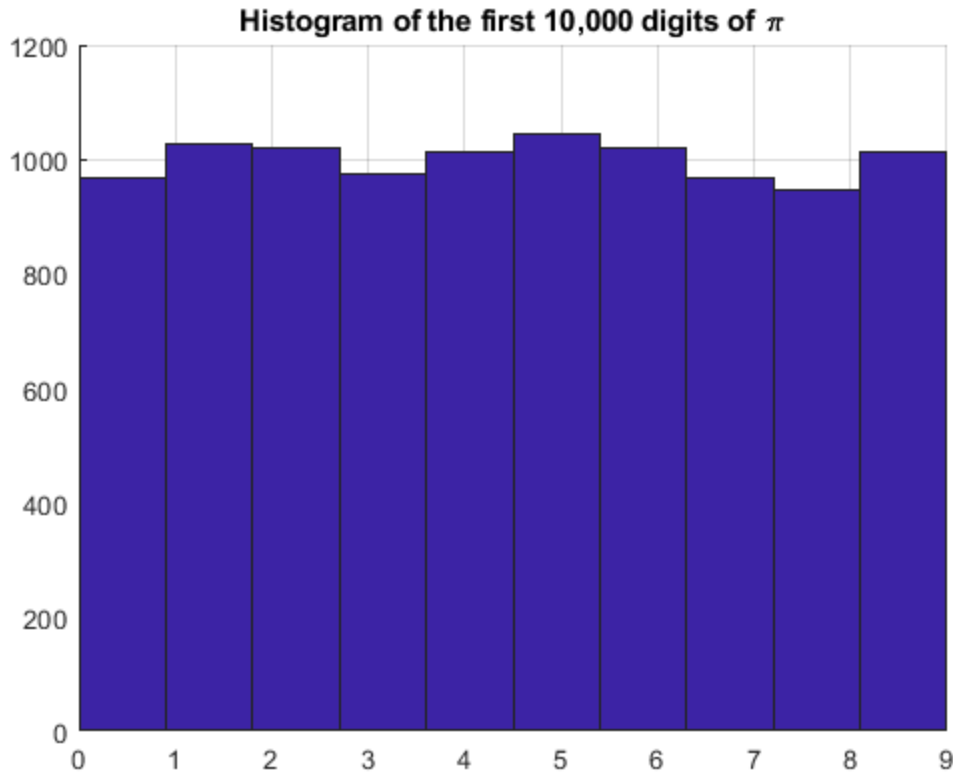
```
% No pattern is apparent. Indeed, the behavior looks pretty random.
```



Part 4: Draw a histogram of how many times each digit is used. Which digit is used the most and which is used the least? How many times are they used?

From part (1) the vector x contains the first 10,000 digits of π . The command `hist(x,n)` will plot a histogram of x , i.e., divide the range of x into n equal intervals and plot a chart showing how many values of x lie in each bin. Since x only takes the 10 integer values $0, \dots, 9$, if we set $n=10$, then each digit will be placed in its own bin.

```
figure;  
hold on;  
grid on;  
title('Histogram of the first 10,000 digits of \pi');  
hist(x,10);
```



The command `h = hist(x,n)` does not draw the histogram, but computes the number of values in each bin and puts these values in the `n`-vector `h`. The last command creates a 2-column vector such that the first column lists the digits 0-9 and the second lists `h`, the number of times that digit occurs.

```
h=hist(x,10);  
[0:9;h]'
```

ans =

0	968
1	1026
2	1021
3	975
4	1012
5	1046
6	1021
7	969
8	948
9	1014

From the values of `h`, we see that the most used digit is 5 (it occurs 1046 times) and the least used digit is 8 (used 948 times).

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