MAT 331 Fall 2023, Homework 2
Volumes of n-balls

(1) Compute the volume of the unit ball in n dimensions using Monte Carlo random sampling in $[0,1]^n$. Do this for $n = 2, 3, \ldots, 20$ and use $1,000,000$ random samples.

(2) Look up the recursive formula for the exact volume of a unit n-ball in Wikipedia. State the formula and use it to compute these volumes for $n = 2, 3, \ldots, 20$.

(3) Present the results of parts (1) and (2) in a 3 column array, where the first column is the dimension, the second is your estimated volume and the third column is the exact volume given by the formula.

(4) Present the same information as two plots in the same figure (the horizontal axis is the dimension and the vertical axis should be the volumes). Plot your estimates with solid line and the exact volumes with a dashed line.

(5) What happens in dimension 20? What volume does the formula give? What estimate do the Monte Carlo random trials give? Can you explain the difference?