Ideas for the Simons Wall Project

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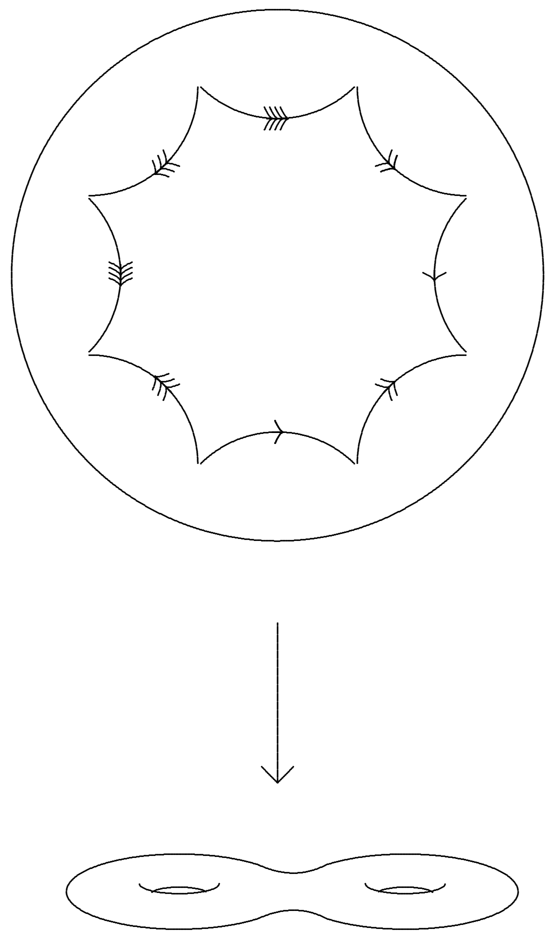
March 5, 2010

Uniformization of a surface of genus 2:



Index theorem for coupled Dirac operator:

$$\not\partial : \Gamma(\mathbb{S}_+ \otimes E) \longrightarrow \Gamma(\mathbb{S}_- \otimes E)$$

$$\dim\ker(\not\partial) - \dim\ker(\not\partial^*) = \int_{M^{4k}} \hat{A}(M) \smile ch(E)$$

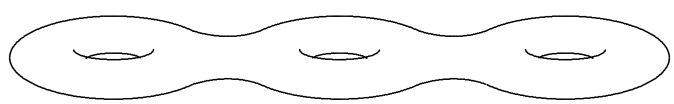
Generalized Gauss-Bonnnet theorem:

$$\chi(M^{2n}) = \frac{1}{(8\pi)^n n!} \int_M \underbrace{R_{ab}^{ij} \cdots R_{cd}^{k\ell}}_{n} \varepsilon^{ab\cdots cd} \varepsilon_{ij\cdots k\ell} \, d\mu$$

Classical Gauss-Bonnnet theorem:

$$\chi(M^2) = \frac{1}{2\pi} \int_M K \, dA$$

$\chi(M) = -4$



Einstein's gravitational field equations:

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Hodge theorem:

$$H^p(M) = \{\varphi \in \Omega^p(M) \mid d\varphi = 0,\ d \star \varphi = 0\}$$

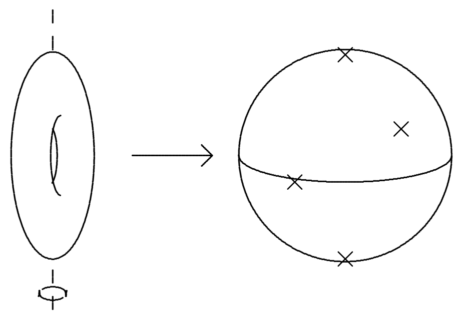
Dirac Equation:

$$
\begin{aligned}
\left(-i\hbar\gamma^{\mu}\nabla_{\mu} + mc\right)\psi &= 0 \\
\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} &= g^{\mu\nu}
\end{aligned}
$$

Seiberg-Witten Equations:

$$\begin{aligned} \partial\!\!\!/_A \psi &= 0 \\ F_A^+ &= -\frac{1}{2}\psi \odot \bar{\psi} \end{aligned}$$

Elliptic curve, and uniformizing elliptic integral:



$$F(z) = \int_{z_0}^{z} \frac{d\zeta}{\sqrt{\zeta(\zeta-1)(\zeta-\lambda)}}$$