

MAT 160 Spring 2010

Homework 2. Due February 9.

1. (from discussion of “Binary 101” card trick).

In “down under” dealing the top card is dealt and the second card put at the bottom of the deck, and so forth.

Prove that if n cards are dealt “down under,” the last card dealt is the card that was originally $\varphi(n)$ from the top, where $\varphi(n)$ is equal to twice the difference between n and the greatest power of 2 which is $< n$.

We checked this for $n = 2^k$: In that case $\varphi(n) = 2(2^k - 2^{k-1}) = 2^k$; as you deal you first deal all the odd-numbered cards, then those numbered of the form $2 \times \text{odd}$, then those numbered of the form $4 \times \text{odd}$; finally the only card left is the one numbered $2^k \times 1$, so the initial bottom card is dealt last.

2. A microbe either splits into two perfect copies of itself or else disintegrates. If the probability of splitting is p , what is the probability that one microbe will produce an everlasting colony?
3. A sequence y_1, y_2, y_3, \dots of real numbers is *monotone* if either $y_1 \leq y_2 \leq y_3 \leq \dots$ or $y_1 \geq y_2 \geq y_3 \geq \dots$. Prove that every sequence of real numbers has a monotone subsequence.
4. Devise an experiment which uses only tosses of a fair coin, but which has success probability $\frac{1}{3}$. Do the same for any success probability p , $0 \leq p \leq 1$.
5. If a set of positive integers has sum n , what is the biggest its product can be?