My NAME is:

Problem	1	2	3	4	Total
Score					

MAT 341
Applied Real Analysis
Midterm 1

October 7, 2011 Total score = 100.

This test is open book: Powers "Boundary Value Problems" may be consulted. No other references or notes may be used. Students may use graphing calculators like TI-83, 84, 85, 86; but they may NOT use calculators with Computer Algebra Systems, like TI-89. SHOW ALL YOUR WORK! When using Powers or your calculator be sure to report it, e.g. "from calculator," "from Powers page x."

1. $(5 + 10 + 10 = 25$ points)

a. Calculate $\displaystyle\int_0^\pi x\cos(5x)\ dx$.

b. Give the solution of the differential equation $\dfrac{dT}{dt} = \alpha T,\ T(0) = T_0$.

c. Give the general solution of the equation

$$\frac{d^2\phi}{dx^2} = -25\phi.$$

2. $(10 + 15 = 25$ points) The function $f(x)$ is defined for $0 \le x \le 1$ by

$$f(x) = \begin{cases} 4x & 0 \le x \le \frac{1}{4} \\ \frac{4-4x}{3} & \frac{1}{4} < x \le 1 \end{cases} .$$

a. Sketch at least two periods of the extension of f to an odd function of period 2.

b. Set up integrals giving the coefficients for the Fourier sine series of f. DO NOT WORK THESE INTEGRALS.

3. (15 + 10 = 25 points) A laterally insulated bar of length π is insulated at $x = \pi$; starting at $t = 0$ the end $x = 0$ is held at temperature 1. The Heat Equation and boundary conditions for this problem are:

$$(*)\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial u}{\partial t} & 0 < x < \pi, \quad 0 < t \\ \\ u(0,t) = 1, \qquad \dfrac{\partial u}{\partial x}(\pi, t) = 0, & 0 < t \\ u(x,0) = f(x). \end{cases}$$

a. What is the steady-state solution $v(x)$ for this problem?

b. Show that setting $w(x,t) = u(x,t) - v(x)$ leads to an initial-value problem with the same partial differential equation as $(*)$ but with homogeneous boundary conditions.

4. (25 points) Consider the eigenvalue problem:

$$\phi''(x) + \lambda^2 \phi(x) = 0, \qquad 0 < x < \pi$$

$$\phi(0) = 0, \qquad \phi'(\pi) = 0.$$

Calculate the first three eigenvalues $\lambda_1^2, \lambda_2^2, \lambda_3^2$ and their associated eigenfunctions ϕ_1, ϕ_2, ϕ_3.

End of Examination.