MY NAME IS:

Problem	1	2	3	4	Total
Score					

MAT 341
Applied Real Analysis
Midterm 1

October 14, 2008

THIS TEST IS "OPEN BOOK" (I.E. POWERS); STUDENTS MAY USE GRAPHING CALCULATORS BUT NOT COMPUTER ALGEBRA SYSTEMS. WHEN REFERENCING POWERS OR USING YOUR CALCULATOR BE SURE TO NOTE THAT FACT.

Explain your answers carefully. Show all your work in the "blue book."

Total score = 100.

1. (30 points) Consider the function defined on $[0, \pi]$ by

$$f(x) = \begin{cases} x & 0 \leq x \leq \frac{\pi}{4} \\ \frac{\pi}{3} - \frac{1}{3}x & \frac{\pi}{4} \leq x \leq \pi \end{cases} .$$

   (a) Set up integrals giving the Fourier sine series for f:

$$f(x) \sim \sum_{1}^{\infty} b_n \sin(nx)$$

   (b) Calculate b_2.

2. (25 points) Solve the heat equation in a laterally insulated bar of length a with end temperatures fixed at 0:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}$$

$$u(0, t) = u(a, t) = 0, \qquad t > 0$$

with initial condition:

$$u(x, 0) = \begin{cases} 1 & \frac{a}{4} \leq x \leq \frac{3a}{4} \\ 0 & \text{otherwise} \end{cases}$$

and sketch the temperature distribution in the bar when $t = 0.1$, assuming $k = 1$ and $a = \pi$.

3. (25 points) Solve the heat equation in a laterally insulated bar of length a

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

with boundary conditions

$$u(0,t) = 0, \qquad u(a,t) = 100 \qquad t > 0$$

and initial condition

$$u(x,0) = 50 \qquad 0 < x < a.$$

4. (20 points) Consider the function

$$f(x) = \begin{cases} -x^2 + x & 0 \le x \le 1 \\ x^2 - 3x + 2 & 1 \le x \le 2 \end{cases}.$$

(a) Show that periodic extension of this function to the whole line is continuous and has a continuous first derivative, but that the second derivative is only sectionally continuous.

(b) Which of the following series can NOT be the Fourier series of f? *Explain your answers!*

i.

$$\frac{4}{\pi^3}[\sin \pi x + \frac{1}{27}\sin 3\pi x + \frac{1}{125}\sin 5\pi x + \cdots]$$

ii.

$$\frac{4}{\pi^3}[\sin \pi x + \frac{1}{81}\sin 3\pi x + \frac{1}{625}\sin 5\pi x + \cdots]$$

iii.

$$\frac{4}{\pi^3}[\cos \pi x + \frac{1}{27}\cos 3\pi x + \frac{1}{125}\cos 5\pi x + \cdots]$$

iv.

$$\frac{4}{\pi^3}[\cos \pi x + \frac{1}{81}\cos 3\pi x + \frac{1}{625}\cos 5\pi x + \cdots]$$

END OF EXAMINATION