NAME:

Stony Brook University - MAT 341 Midterm I
October 11, 2005

Put your name on this sheet and on your blue-book. HAND BOTH IN.

This test is open book: Powers "Boundary Value Problems" may be consulted. No other references or notes may be used.

Students may use graphing calculators like TI-83, 85, 86; but they may NOT use calculators with Computer Algebra Systems, like TI-89.

Total score $= 100$. Show all your work. No credit for unjustified answers!

1. (a) (25 points) Calculate the Fourier series for the function $f(x)$, periodic of period 2, equal to $1 + x^2$ on $[-1, 1]$.

   (b) (10 points) Does this Fourier series converge uniformly? Justify your answer.

2. (a) (15 points) Calculate the steady-state distribution of temperature in a rod of length 10 cm which is laterally insulated, with one end held at $5°$ C, and the other end held at $10°$ C.

   (b) (15 points) Calculate the steady-state distribution of temperature in a rod of length 10 cm which is completely insulated (laterally and at both ends) with initial distribution of temperature given by
   $$u(x, 0) = x^2$$
   where x is measured in cm from the left end.

3. (a) (25 points) Solve the heat equation
   $$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\ln 2} \frac{\partial u}{\partial t}$$
   in a bar of length π, with both ends held at temperature 0, and with initial temperature distribution
   $$u(x, 0) = \sin 2x.$$

   (b) (10 points) Sketch the graph of the temperature distribution at $t = 0$ and at $t = 1$; label your graphs carefully.