Section 9.1

4)

$$\Omega = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 3 \\ 1 & 4 & 1 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$$

a) Applying this to $(1,3,0,2)$ gives $(1,3,1,4)$.

b) Applying it to $(4,1,3,3)$ gives $(1,2,3,0)$.

6) a) The polynomial of interest is $2x^3+2x+1$. We do the left side of the tree first. Dividing the polynomial by x^2-1 leaves a remainder of $2x+1+2x=4x+1$. Dividing this by $x-1$ leaves a remainder of 5, while dividing it by $x+1$ leaves a remainder of -3. We move on to the right side of the tree. Dividing $2x^3+2x+1$ by x^2+1 leaves a remainder of 1. Dividing this by $x-i$ or $x+i$ leaves a remainder of 1.

b) The polynomial of interest is $2x^3+2ix^2-x+i$. We do the left side of the tree first. Dividing the polynomial by x^2-1 leaves a remainder of $-x+i+2x+2i=x+3i$. Dividing this by $x-1$ leaves a remainder of $1+3i$, while dividing it by $x+1$ leaves a remainder of $-1+3i$. We move on to the right side of the tree. Dividing $2x^3+2ix^2-x+i$ by x^2+1 leaves a remainder of $-x+i-2x-2i=-3x-i$. Dividing this by $x-i$ leaves a remainder of $-4i$, while dividing by $x+i$ leaves a remainder of $2i$.

7) x^4-1 factors as $(x^2-1)(x^2-4)$. x^2-1 factors as $(x-1)(x-4)$, and x^2-4 factors as $(x-2)(x-3)$.

8) a) The polynomial of interest is $2x^3+3x+1$. Dividing by x^2-1 yields a remainder of $3x+1+2x=1$. Therefore, dividing by $x-1$ or $x-4$ both yield a remainder of 1. Dividing $2x^3+3x+1$ by x^2-4 yields a remainder of $3x+1+3x=x+1$. Dividing this by $x-2$ leaves a remainder of 3, while dividing it by $x-3$ leaves a remainder of 4.

b) The polynomial is $3x^3+3x^2+x+4$. Dividing by x^2-1 yields a remainder of $x+4+3x+3=4x+2$. Dividing this by $x-1$ leaves 1, and dividing by $x-4$ leaves 3. Dividing $3x^3+3x^2+x+4$ by x^2-4 leaves a remainder of $x+4+2x+2=3x+1$. Dividing this by $x-2$ gives a remainder of 2, while dividing by $x-3$ gives a remainder of 0.

11) $a_{m+N}=a_m$ and $b_{m+N}=b_m$ because $\sin(x)$ and $\cos(x)$ are 2π-periodic. Indeed, since

$$\cos\left(\frac{2\pi(N+m)j}{N}\right) = \cos\left(2\pi j + \frac{2\pi mj}{N}\right) = \cos\left(\frac{2\pi mj}{N}\right),$$

$a_{m+N}=a_m$. A similar computation with sine shows $b_{m+N}=b_m$.

12)

$$\cos\left(\frac{2\pi(N-m)j}{N}\right) = \cos\left(2\pi j - \frac{2\pi mj}{N}\right) = \cos\left(-\frac{2\pi mj}{N}\right) = \cos\left(\frac{2\pi mj}{N}\right)$$

since $\cos(x)$ is an even function. Therefore, $a_{N-m} = a_m$.

$$\sin\left(\frac{2\pi(N-m)j}{N}\right) = \sin\left(2\pi j - \frac{2\pi mj}{N}\right) = \sin\left(-\frac{2\pi mj}{N}\right) = -\sin\left(\frac{2\pi mj}{N}\right)$$

since $-\sin(x)$ is an odd function. Therefore, $b_{N-m} = -b_m$.

Section 9.4

1) a) $f * g = (2, -1, -1, 2, -6, 0)$.

b) $f * g = (0, 0, 3, -2, -11, 22, -20)$.

2) a) $F(f) = (1, 3, 2, 0)$. $F(g) = (2, 3, 0, 4)$.

b) $f * g = (4, 1, 2, 0)$. $F(f * g) = (2, 4, 0, 0)$.

c) Yes, $F(f * g) = F(f)F(g)$, where the multiplication is component-wise on the right-hand side.