Section 9.3

1) a) $|-i| = \sqrt{0^2 + (-1)^2} = 1$. $-i = \cos(3\pi/2) + i\sin(3\pi/2)$.

b) $|\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}| = \sqrt{(\frac{\sqrt{2}}{2})^2 + (\frac{\sqrt{2}}{2})^2} = \sqrt{1/2 + 1/2} = 1$. $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2} = \cos(\pi/4) + i\sin(\pi/4)$.

c) $|-\frac{\sqrt{3}}{2} - i\frac{1}{2}| = \sqrt{(-\frac{\sqrt{3}}{2})^2 + (-\frac{1}{2})^2} = \sqrt{3/4 + 1/4} = 1$. $-\frac{\sqrt{3}}{2} - i\frac{1}{2} = \cos(7\pi/6) + i\sin(7pi/6)$.

2)

$$\omega^3 = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2\left(i\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right)\left(i\frac{\sqrt{3}}{2}\right)^2 + \left(i\frac{\sqrt{3}}{2}\right)^3 = \frac{1}{8} + i\frac{3\sqrt{3}}{8} - \frac{9}{8} - i\frac{3\sqrt{3}}{8} = -1.$$

$$\omega^6 = (\omega^3)^2 = (-1)^2 = 1$$

Fourier Series Problems

We must choose a convention for the value of $f(\pi)$. This will not affect the actual values of the Fourier coefficients, but it will affect the values of the approximations. We will take $f(\pi) = 1$ so that

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le \pi \\ -1 & \text{if } \pi < x \le 2\pi \end{cases}$$

We can now calculate

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx = 0$$

$$a_m = \frac{1}{\pi}\left(\int_0^{\pi} \cos(mx)dx + \int_{\pi}^{2\pi} -\cos(mx)dx\right) = \frac{1}{m\pi}\left(\sin(m\pi) - \sin 0 - \sin(2\pi m) + \sin(m\pi)\right) = 0$$

$$b_m = \frac{1}{\pi}\left(\int_0^{\pi} \sin(mx)dx + \int_{\pi}^{2\pi} -\sin(mx)dx\right) = \frac{1}{m\pi}\left(-\cos(m\pi) + \cos 0 + \cos(2\pi m) - \cos(m\pi)\right) = \frac{2}{m\pi}(1-(-1)^m)$$

Therefore, $b_m = 0$ if m is even, and $b_m = \frac{4}{m\pi}$ if m is odd. $f(x) = \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi}\sin(2n+1)x$.

The approximate values of the first few coefficients, as given by the Left-Hand Riemann sum with 100 rectangles, are as follows:

$a_0 \approx .02$. $a_1, a_3, a_5, a_7 \approx 0$. $a_2, a_4, a_6, a_8 \approx .04$. $b_n \approx 0$ for n even. All of these coefficients should be exactly 0.

$b_1 \approx 1.2728$, while the exact value is $4/\pi \approx 1.2732$.

$b_3 \approx .4232$, while the exact value is $4/3\pi \approx .4244$.

$b_5 \approx .2526$, while the exact value is $4/5\pi \approx .2546$.

$b_7 \approx .1789$, while the exact value is $4/7\pi \approx .1819$.