

MAT 312/AMS 351 – Fall 2010
Homework 7

1. Explain why if $n > 2$ then $\varphi(n)$ is even.
2. Fill in this table according to the map $\mathbf{Z}_5^* \times \mathbf{Z}_{12}^* \rightarrow \mathbf{Z}_{60}^*$ calculated from the algorithm in the proof that $\varphi(ab) = \varphi(a)\varphi(b)$. Show your work. Note that vertically this table will not have the same regularity as the one done in class for $\mathbf{Z}_5^* \times \mathbf{Z}_8^*$.

	$[1]_5$	$[2]_5$	$[3]_5$	$[4]_5$
$[1]_{12}$				
$[5]_{12}$				
$[7]_{12}$				
$[11]_{12}$				

3. The *multiplicative order* of the non-zero congruence class $[a]_n$ is by definition the smallest $k > 0$ such that $a^k \equiv 1 \pmod n$. Exercise 1 p. 54.
4. Exercise 2 p. 54.
5. Exercise 3 p. 54.
6. The *additive order* of the congruence class $[a]_n$ is by definition the smallest $k > 0$ such that $a + a + \dots + a$ (k times) $\equiv 0 \pmod n$. Calculate the additive order of all the congruence classes mod 12.
7. Explain why $[a]_{12}$ and $[12 - a]_{12}$ have the same additive order.