MAT 312/AMS 351 – Fall 2010
Homework 4

1. p. 43 Exercise 1.

2. p. 43 Exercise 3. Consider also the numbers 9 and 11.

3. Given positive integers a and b, suppose there exist integers k and ℓ such that $1 = ka + \ell b$. Show that a and b must be relatively prime.

4. Suppose a positive number $n > 1$ is relatively prime to every number strictly between 1 and n. Show that n must be a prime.

5. Using the previous 2 exercises, or otherwise, prove that in the number system of integers $mod\ n$, if every class has a multiplicative inverse then n must be a prime.

6. Write out the multiplication table for the set of congruence classes mod 11.

7. For each nonzero class $x = [1], [2], \ldots, [10]$, write out the sequence of powers of x mod 11. For example $[5]^1 = [5], [5]^2 = [25] = [3], [5]^3 = [3 \cdot 5] = [15] = [4]$, etc. Stop when you get to $[1]$.

8. The number of distinct powers of x (counting $x^0 = [1]$) is called the (multiplicative) *order* of x. What are the different orders occurring in your list? What numerical property do these orders share?