Stony Brook University
MAT 310 Linear Algebra Fall 2008
Review for Final Examination

All material in Reviews for Midterms 1 and 2, along with the following.

(References to Friedberg *et al.*, Linear Algebra, 4th Ed.)

5.1 Be able to compute eigenvalues and eigenvectors for a linear operator $T: V \to V$, directly as in *Examples 1,2* or using the characteristic polynomial (*Example 6*). Understand that an eigenvector is only determined up to multiplication by a non-zero scalar (see Example 6 and discussion of Example 7). *Exercises 3b,c 9, 11c.*

5.2 Be able to prove Theorem 5.5: eigenvectors corresponding to distinct eigenvalues are linearly independent. Understand what it means for a polynomial with coefficients in a field $\mathbf{F}$ to *split over* $\mathbf{F}$. Understand what the multiplicity of an eigenvalue is. Understand the concept of *eigenspace*, and be able to prove the analogue of Theorem 5.5 for eigenspaces (the Lemma on page 267). Understand why (Theorem 5.9) T is diagonalizable if and only if the multiplicity of each eigenvalue is equal to the dimension of the corresponding eigenspace. *Examples 6, 7.*

5.4 Understand what a T-invariant subspace is, for $T: V \to V$ a linear operator. *Example 1.* Know the definition of the T-cyclic subspace of V generated by $v \in V$ (page 313), and be able to prove this subspace is T-invariant. Be able to prove that, if T_W is the restriction of T to the T-invariant subspace W, then the characteristic polynomial of T_W divides the characteristic polynomial of T; *Example 5.* Be able to compute the characteristic polynomial of the restriction of T to a T-cyclic subspace (Theorem 5.22); *Example 6.* Understand the proof of the Cayley-Hamilton Theorem and the C-H Theorem for Matrices; *Example 7. Exercises 3, 6a, 9, 10, 15.*

6.1 Be able to check whether a function $\phi: V \times V \to \mathbf{F}$ is an inner product (V is an $\mathbf{F}$-vector space, $\mathbf{F} = \mathbf{R}$ or $\mathbf{C}$). Understand the conjugate transpose A^* of a matrix A (*Example 5* and the inner-product space H defined on p. 332). Be able to use the Cauchy-Schwartz and Triangle inequalities (Theorem 6.2) and understand their derivation. Be able to derive the inner product from the norm *Exercise 20a* when $\mathbf{F} = \mathbf{R}$. *Example 9.*

6.2 Be able to carry out the Gram-Schmidt orthogonalization process (Theorem 6), and be able to prove Corollary 1 (the components of a vector y in an orthonormal basis are the inner products of y with the basis elements) *Example 3. Exercise 15a* is important.

6.3 Understand the definition of the adjoint T^* of an operator $T: V \to V$ on an inner-product space, and that the matrix in an orthonormal basis of T^* is the conjugate transpose of the matrix of T (Theorem 6.10) *Example 2.*