My NAME is: in Section .

Problem	1	2	3	4	5	Total
Score						

MAT 310 Linear Algebra Midterm 1

October 13, 2008

No books or notes may be consulted during this test.
Explain your answers carefully. Show all your work in the "yellow book."
Total score = 100. Each part of each question is worth 10 points.

1. (a) In the vector space $\mathbf{R}^4$, is the vector $(-1,1,1,2)$ in the span of the vectors $(1,0,1,-1)$ and $(0,1,1,1)$?

   (b) In the vector space $M_{2\times 2}(\mathbf{R})$, i.e. 2 by 2 matrices with real entries, addition and scalar multiplication defined AS USUAL by

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix},$$

$$c \cdot \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{pmatrix},$$

   is the set

$$\left\{ \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ -4 & 4 \end{pmatrix} \right\}$$

   linearly independent?

2. (a) Prove that in $\mathbf{R}^4$ the set S of vectors $\mathbf{v} = (v_1, v_2, v_3, v_4)$ satisfying $3v_1 + v_2 - v_3 - 5v_4 = 0$ is a subspace.

   (b) Prove that S has dimension 3.

3. (a) Let $\mathcal{C}([0,2])$ represent the vector space of continuous functions defined on the interval $[0,2]$, and consider the function $T\colon \mathcal{C}([0,2]) \to \mathbf{R}$ given by

$$T(f) = \int_0^1 f(x)\,dx - \int_1^2 f(x)\,dx.$$

   Is T a linear transformation? Explain in detail.

   (b) With $M_{2\times 2}(\mathbf{R})$ as above, let $S\colon M_{2\times 2}(\mathbf{R}) \to M_{2\times 2}(\mathbf{R})$ be defined by $S(A) = 2A+I$, where I is the 2×2 identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

   Is S a linear transformation? Explain in detail.

4. The linear transformation $T\colon \mathbf{R}^4 \to \mathbf{R}^4$ given with respect to the standard basis (in both $\mathbf{R}^4$s) by the matrix

$$\begin{pmatrix} 4 & 3 & 2 & 1 \\ -1 & 0 & 1 & -1 \\ 3 & 1 & -1 & 2 \\ -2 & -2 & -2 & 0 \end{pmatrix}$$

has rank 2 and nullity 2.

(a) Give a basis for the range of T.

(b) Give a basis for the null-space of T.

5. Given matrices

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & -1 \\ -2 & 1 \\ 1 & -1 \end{pmatrix}.$$

(a) Calculate the matrix product AB.

(b) Calculate the matrix product BA.

END OF EXAMINATION