MAT 200
SOLUTIONS TO HOMEWORK 8

NOVEMBER 16, 2004

Geometry notes: Exercises 8.4, 8.6

8.4 Existence: let m be a line passing through A and perpendicular to $\overleftrightarrow{OA}$ (such a line exists by Protractor axiom). By Proposition 8.8, m is a tangent line to the circle.

Uniqueness: assume that m_1, m_2 are two tangent lines passing through A. Then, by Proposition 8.9, both m_1, m_2 are perepependicular to $\overleftrightarrow{OA}$. But by protractor axiom, this implies that $m_1 = m_2$.

8.6 By Proposition 8.9, $\overleftrightarrow{OA} \perp k$ and $\overleftrightarrow{OB} \perp m$. But since $k \parallel m$, by Proposition 6.3, we also have $\overleftrightarrow{OB} \perp k$. Thus, $\overleftrightarrow{OA}, \overleftrightarrow{OB}$ are two perpendiculars from O to k. Since the perpenidicular is unique (Theorem 6.4), this implies $\overleftrightarrow{OA} = \overleftrightarrow{OB}$, so points O, A, B lie on a single line. Thus, $\overleftrightarrow{AB}$ passes through O.

Section 5.1: 2 a-c, 5

2 a-c (a) $\{n^2 \mid (n \in \mathbb{N}) \wedge (n \leq 100)\}$
(b) $\{n^2 \mid n \in \mathbb{N}\}$
(c) $\{(-2)^n \mid n \in \mathbb{N}\}$

5 Let x, t be real variables. Then we may write A as $A = \{x + 3 | x = \tan x\}$.

Let us make change of variables $t = x + 3$, so that $x = t - 3$. Then we can rewrite $A = \{t | t - 3 = \tan(t - 3)\}$.

Answer : c)