MAT 200
SOLUTIONS TO HOMEWORK 3

SEPTEMBER 28, 2004

Section 3.2: 3, 6, 8 a-f

(3) (a) $\sim\exists n\ (n > 0 \wedge n < 1)$

(b) $\sim\exists m\ \forall n\ (n \leq m)$

(c) $\exists n\ (2n + 1 = m)$ (for future use, denote it by $odd(m)$)

(d) $\sim\exists m\ (m > 1 \wedge m < n \wedge (\exists k(mk = n)))$ (for future use, denote this by $prime(m)$

(e) $\forall n\ [(prime(n) \wedge (n \neq 2)) \to odd(n)]$

(f) $\forall n\ [prime(n) \to \exists m\ (prime(m) \wedge m > n)]$

(g) $\forall x\ \Big((\sim\exists n\ (n = x)) \to (\exists n\ (x < n < x + 1))\Big)$

(h) $\forall x\ \forall y\ \Big((x \neq y) \to \exists z\ (x < z < y \vee y < z < x)\Big)$

(6) (a) "For all real x, $x \geq 0$ implies that there exists a real number y for which $y^2 = x$." Better: "Every nonnegative real number has a square root."

(b) "For all real x, $x \leq 0$ implies that there does not exists a real number y for which $y = \log[x]$." Better: "If $x \leq 0$, $\log x$ does not exist in the set of real numbers."

(c) There exists a number x so that, for any y we have $x \cdot y = y$. (This number is generally called 1.)

(d) "For all real a and b, if a is non-zero, then there is an x for which $ax + b = 0$." Better: "Any line which is not horizontal intersects the x-axis."

(8) (a) Let p: asparagus s: Spinach h: human
$L(x, y)$: x likes y.
Then the answer is

$$\sim\forall h\, L(h, s) \wedge \ \sim\exists h\, L(h, a)$$

or

$$\sim\forall h\, L(h, s) \wedge \forall h\ \sim L(h, a)$$

(b) Be careful of the fact that the two 'are's have different meaning!
Let x: something $C(x)$: x is a crow $B(u)$: u is black.

$$\{\forall x(C(x) \to B(x))\} \wedge \{\exists x(B(x) \wedge \ \sim C(x))\}$$

(c) p: person f: frog $K(x, y)$: x kisses y $B(x)$:x benefits.

$$(\exists p\, K(p, f)) \to (\forall p\, B(p))$$

(d) p: person v: vegetable $L(x, y)$: x likes v.

$$\exists p\, (\forall v\, L(p, v))$$

(e) p: person t: time $F(x, t)$: It's possible to fool x at time t.

$$\{\exists t\ (\forall p\, F(p, t))\} \wedge \{\forall t\ (\exists p\, F(p, t))\} \wedge \ \sim\{\forall t\ (\forall x\, F(x, t))\}$$

(f) m: myself p: person (other than me) $B(x, y)$: x bothers y $H(x, y)$: x helps y.

$$(\forall p\, B(p, m)) \to (\forall p\ \sim H(m, p))$$

Section 3.3: Problems 5, 7a-c, 9 a,b,d

(5) (a) True, all numbers have a square.
(b) False, negative numbers have no real square root.
(c) False, but $\forall y \; \exists x (x + 5 = y)$ would have been true.
(d) False. Statement $\forall u \; x + z = y + u$ is false regardless of values of x, y, z.
(e) True, $x^2 + y^2$ is necessarily non-negative and hence has a square root.
(f) True, choose $x = -1$.

(7) (a) Not a law of logic. Having one case of x where $P(x)$ holds does not imply that $P(x)$ holds for all x.
(b) Yes, this is a law of logic. This could be argued intuitively, but here is the formal way to argue :
If $\exists x \; \forall y \; P(x, y)$ is true, we have a value of x, say a, such that $P(a, y)$ is true for all y. Thus, for any y, there is a value of x for which $P(x, y)$ is true — namely, $x = a$. So $\forall y \; \exists x \; P(x, y)$ is true.
(c) Not a law of logic. We can see this from example 3 and 4 in 3.3 of the textbook.

(9) (a) $\forall x \in A \; \exists y > 0 \; y^2 = x$
(b) $\forall x (\exists n \; n > x) \wedge (\exists m \; m < x)$ (here x is a real variable and m, n are integer variables).
(d) $\sim (\exists x > 0 \; \exists y < 0 \; x = y)$, i.e. "it is not true that there exist a positive number and a negative number which are equal". Can also be written as $\forall x > 0 \; \forall y < 0 \; x \neq y$.