MAT 200
SOLUTIONS TO HOMEWORK 2

SEPTEMBER 21, 2004

CORRECTED FEBRUARY 17, 2005

Section 2.3: Problems 4 a,b,e; 5 a-c

(4) (a) Valid.

(b) Valid.

(e) Valid.

Let P: Turtles can sing; Q: Artichokes can fly. R: Dogs can play chess.

If we make a truth table of the statement

$$[(P \to Q) \wedge \{Q \to (P \wedge \sim R)\} \wedge (R \leftrightarrow P)] \to \sim P.$$

we find that it is a tautology.

(5) (a)

(1)	$Q \to R$	Premise
(2)	$R \vee S \to P$	Premise
(3)	$Q \vee S$	Premise
(4)	$\sim S \to Q$	From step 3, by tautology 20
(5)	$\sim S \to R$	From steps 4 and 1, by Transitivity of implication
(6)	$S \vee R$	From step 5, by tautology 20
(7)	P	From steps 6 and 2, by Modus ponens.

(b)

(1)	$P \to (Q \leftrightarrow \sim R)$	Premise
(2)	$P \vee \sim S$	Premise
(3)	$R \to S$	Premise
(4)	$\sim Q \to \sim R$	Premise

Informal proof: proof by conradiction. Assume R is true; then, by (4), Q is true and by (3), S is true. Thus, by (2), P is true, so $Q \leftrightarrow \sim R$. But this contradicts the fact that both Q and R are true. Thus, our assumpiton is false, so R must be false.

Here is the same in a more formal way:

(1)	$P \to (Q \leftrightarrow \sim R)$	Premise
(2)	$P \vee \sim S$	Premise
(3)	$R \to S$	Premise
(4)	$\sim Q \to \sim R$	Premise
(5)	Assume R	
(6)	| Q	(4), (5), Modus Tollens
(7)	| S	(3), (5), Modus Ponens
(8)	| P	(2), (7)
(9)	| $Q \leftrightarrow \sim R$	(1), (8), Modus Ponens
(10)	| contradiction	(5), (6), (9)
(11)	$\sim R$	(5)–(10), proof by contradiction

(c) **Premises:** "Babies are illogical." "A person who can manage a crocodile is not despised." "Illogical persons are despised." [NOTE TYPO IN BOOK]

Argument:
"Babies are illogical" $\wedge$ "Illogical persons are despised" $\rightarrow$ "Babies are despised" (Because babies are persons)
"A person who can manage a crocodile is not despised" $\rightarrow$ "A person who is despised cannot manage a crocodile " (contrapositive)
"Babies are despised" $\wedge$ "A person who is despised cannot manage a crocodile " $\rightarrow$ "Babies cannot manage a crocodile" (transitivity of implication)

Section 4.2: 4, 5, 13, 15, 16

(4) (a) I cannot make you happy. (Modus Tollens)
(b) I am very sad. (Modus Ponens twice, or tranistivity of implication and Modus Ponens used once — see next problem).
Transitivity of implication would give $P \rightarrow R$; then we need to use Modus Ponens to get R
(c) I'll come out ahead in my bets. (Proof by Cases.)
(d) This function is not continuous. (Modus Tollens.)

(5) (a) H: I make you happy today. P: I am in two places at once.

(1)	$H \rightarrow P$	Premise
(2)	$\sim P$	Premise
(3)	$\sim H$	(1), (2), Modus Tollens

(b) S: it is Saturday. L: I go to school. D: I am very sad

(1)	$S \rightarrow \ \sim L$	Premise
(2)	$\sim L \rightarrow D$	Premise
(3)	S	Premise
(4)	$\sim L$	(1), (3), Modus Ponens
(5)	D	(4), (2), Modus Ponens

(c) W: Mets win. B: I come ahead on my bets.

(1)	$W \rightarrow B$	Premise
(2)	$\sim W \rightarrow B$	Premise
(3)	$W \vee \ \sim W$	Tautology
(4)	B	(1), (2), (3), Proof by cases

(d) C: function is continuous. I: function is integrable.

(1)	$C \rightarrow I$	Premise
(2)	$\sim I$	Premise
(3)	$\sim C$	(1), (2), Modus Tollens

(13) This proof is wrong. When doing proof by cases, you need to show that at least one of the cases always holds, so your cases cover all possibilities. This proof doesn't consider the case of $0 < x < 1$. Indeed, for $x = \frac{1}{2}$, $x^2 = \frac{1}{4}$, so clearly in this case $x > x^2$.

(15) This proof is fine, though it should be noted that in general you cannot multiply by $1/x$. The fact that it is assumed that $x \neq 0$ however allows us to multiply by $1/x$.

(16) This proof is wrong. Indirect proof requires us to negate the proposition. If the proposition is $P \rightarrow Q$, then its negation is $P \wedge \ \sim Q$ (by tautology 19), so the indirect proof should assume that n is even and n^2 is not even. The given proof assumes that n^2 is even and n is not even.

The proposition itself is true. Here is the correct proof: assume that n is even. Then $n = 2k$ for some k, so $n^2 = (2k)^2 = 2 \cdot 2k^2$ is even. Thus, by conditional rule, (n is even $\rightarrow$ n^2 is even).