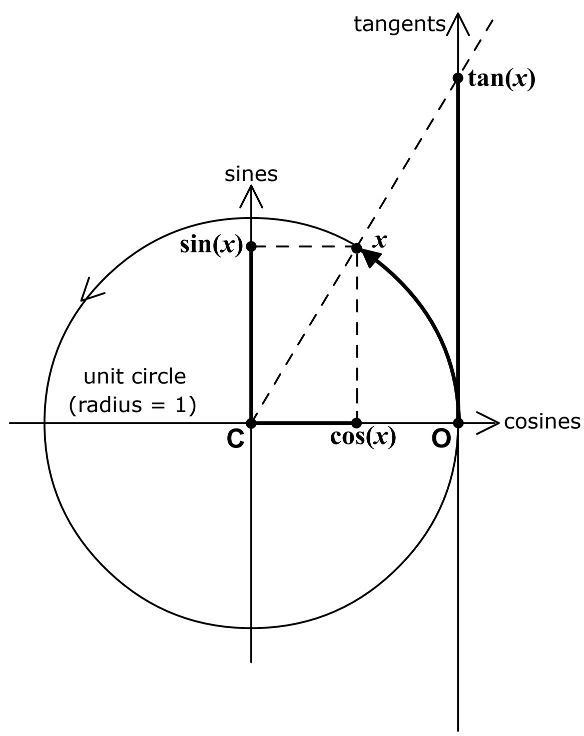
MAT125 Fall 2013 Review for Midterm I

1.1 Understand the definition of function (box, p. 12) and the importance of the word "exactly." The Vertical Line Test (p. 17) shows exactly what "exactly" means in terms of graphs (*Exercises* 5-8). Be able to ascertain the domain of a function either from the formula (*Example* 6, *Exercises* 29-33) or the graph (*Example* 1, *Exercises* 1, 2, 5-8). Practice sketching graphs (*Exercises* 36-39), especially for piecewise-defined functions (*Exercises* 43-46).

1.2 Be familiar with the "essential functions." Linear (*Example* 1, *Exercises* 10, 11, 15, 18); be able to interpret slope and y-intercept. Polynomial: know what the degree is (p. 29), (*Example* 4, *Exercise* 3). Power functions: understand that the $\frac{1}{n}$ power (n a positive integer) corresponds to the nth root; and that if n is even, $f(x) = x^{1/n}$ is only defined for positive x; understand negative powers as reciprocals of the positive: $x^{-n} = 1/x^n$; be comfortable with the law of exponents in this context: $x^a x^b = x^{a+b}$, $x^a/x^b = x^{a-b}$, $(x^a)^b = x^{ab}$; a and b can be integral, fractional, positive, negative, anything.



Trigonometric functions: keep this diagram firmly and permanently in mind to understand sine, cosine and tangent as functions of x in radians. Starting at O (the right-hand intersection of the unit circle with the horizontal axis), go a distance x counterclockwise along the circle. Projection onto the horizontal (cosines) axis then gives $\cos(x)$; projection onto the vertical (sines) axis gives $\sin(x)$; the line through C (center of the circle) and x intersects the tangents axis at $\tan(x)$. (The tangents axis is tangent to the circle at O.)

Elementary properties of $\sin, \cos, \tan$ can be retrieved from this diagram, e. g. $-1 \leq \sin(x) \leq 1$, $-1 \leq \cos(x) \leq 1$, $\tan(x) = \sin(x)/\cos(x)$ (similar triangles), $\sin^2(x) + \cos^2(x) = 1$ (Pythagorean theorem), $\sin(x + 2\pi) = \sin(x)$, $\cos(x + 2\pi) = \cos(x)$ (2π is length of circle), etc.

Exponential and logarithmic functions, also "essential," are covered in §1.5.

1.3 Understand how translations work (Box, p. 38 and Figure 1); remember that $f(x-c)$ has the graph of $f(x)$ shifted c units to the *right*, and understand why! (*Exercise* 3ade). Same for stretching and reflecting (Box, p. 39 and Figure 2); remember that for example $f(2x)$ has the graph of $f(x)$ *compressed* by a factor of 2, and understand why. Also understand why $-f(x)$ and $f(-x)$ do very different things to $f(x)$. *Exercises* 1, 2, 3, 4, 5.

Understand that when two functions are combined by $+, \times, -, \div$ the domain of the combination is the *intersection* of the two domains: the set of points where both functions are defined, plus no zeroes in the denominator! *Exercises* 29, 30. Be able to "complete the square" (*Example* 2, *Exercise* 12). Remember that in the composition $f(g(x))$ the function g is applied first. (*Example* 6, *Exercises* 31-36).

1.5 Be able to sketch the graph of an exponential function $f(x) = a^x$, for any *positive* number a: $f(0) = 1$ always. The function increases from 0 to infinity if $a > 1$, it's constant and equal to 1 if $a = 1$, it decreases from infinity to 0 if $0 < a < 1$. *Only positive a are considered!* (*Exercises* 7, 8, 9). Be familiar and completely comfortable with the Laws of Exponents (box, p. 54). Know what e is (p. 57). (*Example* 4, *Exercises* 14, 15, 16).

1.6 Understand that a function can only have an inverse if it is one-one, i.e. it satisfies the Horizontal Line Test (box, p. 61). *Examples* 1, 2 are fundamental. *Exercises* 5-8. Understand and be able to apply the algorithm (box, p. 64) to calculate the inverse of a one-one function. (*Example* 4, *Exercises* 21, 22).

In particular the exponential function $f(x) = a^x$ for any positive *a not equal to* 1 has an inverse, called "logarithm to the base a," and written $\log_a$. Understand and be comfortable with the equivalences between $\log_a x = y$ and $x = a^y$, etc. (Boxes on p. 65!!). Be comfortable with the Laws of Logarithms (Box, p. 65). (*Example* 6, *Exercises* 35,36). Know the natural logarithm $\ln x = \log_e x$, and know the change-of-base formula $\log_a x = (\ln x)/(\ln a)$ (Box, p. 67. *Example* 10 -requires a calculator).

2.1 Be able to sketch secant lines through a point on a curve, and the tangent line to the curve at that point, and understand how to use slopes of secants to *estimate* the slope of the tangent (*Examples* 1, 2, *Exercises* 3, 4 -require calculator). Be able to calculate average velocities from distance-time data and to use them to estimate instantaneous velocity (*Exercises* 6, 7).

Use the Chapter Reviews for further reviewing.

Chapter 1

- Concept Check 1 through 12.
- True-False 1 through 11.
- Exercises 1 through 16, 18,19,20.

9/16/13