MAT125 Fall 2007 Review for Midterm II

2.5 Understand how $\lim_{x\to a} f(x) = \infty$, etc., give a *vertical asymptote* at a (Box 2 p.129, Example 1 p.130). Exercises 3, 7. Understand how $\lim_{x\to\infty} f(x) = L$, etc., give as *horizontal asymptote* the line $y = L$ (Box 5 p.132, Examples 3,4 p.133). Exercises 3, 7.

Be able to calculate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the special case where $f(x) = p_1(x)/p_2(x)$ is the quotient of two polynomials, p_1, p_2. Use the "divide by the highest power of x in the denominator" method (explained on p.134). Example 5 p.135, Example 10 p.137. Exercises 21-23.

2.6 Understand that the slope of the tangent line is the limit of the slopes of secant lines (Figure 1, Box 1, p.140). Be comfortable with both notations: $x \to a$ and $h \to 0$, where $h = x - a$. (Compare Box 1 and Box 2, p.141). Example 1 p.140, Example 2 p.142. Exercises 7,8. Understand also that instantaneous velocity at $t = a$ is the limit of average velocities over smaller and smaller time periods beginning or ending with a. Box 3 p.142. Example 3. Exercises 17, 18ab.

2.7 Basic concept: the derivative of f at a (Box 2 p.148). Understand that the derivative of f at a is the slope of the line tangent to the graph of f at the point $(a, f(a))$ (Box, Example 2, p.149). Exercises 4, 7. Also understand that the derivative of f at a is the instantaneous rate of change of f at a; if $f(t)$ is position as a function of time t, then $f'(t)$ is instantaneous velocity at time t. Example 4. Exercises 25, 26.

2.8 Be able to sketch the graph of f' given the graph of f. Example 1 p.156. Be able to calculate $f'(x)$ from the definition in simple cases (Examples 3, 4, 5 pp.158-159), Exercises 19-22. Understand why $f(x) = |x|$ does not have a derivative at $x = 0$ (Example 6 p.160). Understand how to calculate the second derivative $f''(x)$ (Example 7 p.163) and its interpretation in terms of acceleration (Example 8 p.164; Exercise 38).

2.9 Be able to tell by examining f' where f is increasing and where it is decreasing (Box, p.169; Example 1, Exercises 1,2). Be able to tell from f'' where the graph is concave upward and where it is concave downward (Box, p.170; Example 2, Exercise 8).

3.1 Know the elementary differentiation rules: $\frac{d}{dx}(c) = 0$ and $\frac{d}{dx}(x) = 1$ (Boxes, p.183) and understand what these equations mean in terms of slopes. Know the *Power Rule*: $\frac{d}{dx}(x^n) = nx^{n-1}$ (Boxes, p.184 and p.185). Be familiar with the special cases $n = \frac{1}{2}$ ($f(x) = \sqrt{x}$) and $n = -1$ ($f(x) = \frac{1}{x}$). Examples 2, 3. Be able to calculate the derivative of $rf(x) + sg(x)$ for constants r, s knowing the derivatives of f and g separately. (Boxes, pp.186-187 Examples 4, 5).

Know how to differentiate the "natural exponential function" $f(x) = e^x$ (Box, p.190; Example 8, Exercises 10, 29).

3.2 Be able to apply the product and quotient rules correctly (Box, p.194; Examples 1a, 2, 3; Exercise 13). (Box, p.197; Examples 5, 6; Exercises 11, 19). If you can't remember where the minus sign goes in the quotient rule, use $\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$ to check.

3.4 Be able to sketch the graphs of $\sin x$ and $\cos x$ to scale ($\pi = 3.14..$) and to convince yourself that $\sin' = \cos$ and that $\cos' = -\sin$. (Boxes 4, 5 pp.215-216; Example 1 p.216; Exercises 3, 4, 6, 7). Remember or be able to calculate that $\tan' = \sec^2$. [This is a good place to review your trigonometric identities, especially $\sin^2 + \cos^2 = 1$ and $\tan^2 + 1 = \sec^2$.] Example 2.

3.5 Know and be comfortable with the Chain Rule in both forms (Box, p.220). Example 1, p.221: read the Note at the end. Practice as many examples as you can. Special case with the "outside function" a power: Box 4 on p.223; Examples 3, 4, 5. Be able to apply the Chain Rule to $f(x) = a^x = e^{x \ln a}$ to obtain $f'(x) = a^x \ln a$ (Box 5, p.224; Exercise 20).

3.6 Implicit differentiation always involves the chain rule. In a case like Example 1 p.233, where we are differentiating with respect to x, the derivative of x^2 is $2x$ but the derivative of y^2 is $2y\frac{dy}{dx}$. The pattern is always the same: take $\frac{d}{dx}$ of *everything*, then solve for $\frac{dy}{dx}$ (Example 2; Exercises 3, 4, 5).

Be able to use implicit differentiation to calculate the derivatives of the inverse trigonometric $\sin^{-1}$ and $\tan^{-1}$ (Boxes, p.237). [Need those trig identities.] Exercises 29,30.

3.7 Understand why $\frac{d}{dx}\ln x = \frac{1}{x}$, and be able to use this fact with the Chain Rule as in Examples 1, 2, 3 on p.341 (Exercises 2, 3, 6). Know how and when to apply *logarithmic differentiation* (Box, p.243; Example 8; Exercises 27-30).

Use the Chapter Reviews for further reviewing.

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