MAT 126: PROBLEM SET 3

1. Recalling the identity

$$\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x),$$

evaluate the following integrals:

   (a) $\int \sin^2(x)\,dx$

   (b) $\int \cos^2(x)\sin^2(x)\,dx$

   (c) $\int \cos^4(x)\,dx$

2. For these problems, do not use the double angle formulas. Instead, make a substitution and rewrite everything in terms of only one trig function.

   (a) $\int \sin^2(x)\cos(x)\,dx$

   (b) $\int \sin^5(x)\,dx$

   (c) $\int \sin^4(x)\cos^7(x)\,dx$

3. To be solved by reverse trigonometric substitutions:

   (a) $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx$

   (b) $\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

   (c) $\int \frac{x^2}{\sqrt{2x-x^2}}\,dx$ HINT: *First complete the square.*

   (d) $\int \frac{1}{(4x^2-24x+27)^{3/2}}\,dx$ HINT: *Ditto.*

4. To be solved using the "*magic*" substitution $u = \tan(x/2)$:

   (a) $\int \sec(x)\,dx$

   (b) $\int \frac{5}{3\sin(x)+4\cos(x)}\,dx$

   (c) $\int \frac{1}{7\cos(x)-\sin(x)+5}\,dx$

5. Find the derivatives of the following functions:

(a) $F(x) = \int_1^x t^2 \sin(t)\, dt$

(b) $F(x) = \int_{-x}^{\log(x)\cos(x)} e^t \arctan(t)\, dt$

(c) $F(x) = \int_2^{\int_0^x \arctan(t)\, dt} \tan(t)\, dt$