By law of conservation of What is wrong? energy, this should not work.

Consider any shell S bounding domain B with internal pressure P.

 $\frac{P}{h} = \frac{B}{5}$ What is the total force \vec{F} on the shell by the pressure P. The force \vec{F} is

$$\vec{F} = \iint \vec{P} \cdot \vec{n} \cdot dS = P \iint \vec{n} \cdot dS$$

Since P is constant.

Note

$$\vec{F} = P \iint_{S} \vec{n} \, dS$$

$$\vec{i} \cdot \vec{F} = P \iint_{S} \vec{n} \cdot \vec{i} \, dS$$

$$\vec{i} = (1,0,0)$$

$$S$$

$$= P \iint_{S} div \vec{i} \, dV = 0 \quad as \quad div \vec{i} = 0$$

$$B$$

Also

$$\vec{j} \cdot \vec{F} = 0$$
 and $\vec{k} \cdot \vec{F} = 0$.
Thus $\vec{F} = 0$, there is no force!
Maybe there exists a total torque,
so that the construction will rotate?
 $\vec{k} \cdot \vec{F} = 0$.

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The torque is

$$\vec{T}_{r} = \vec{T} \times \vec{F} \qquad \vec{T}_{r} = \vec{T} \times \vec{T} \qquad \vec{T}_{r} = \vec{T} \qquad \vec{T} \qquad \vec{T}_{r} = \vec{T} \times \vec{T} \qquad \vec{T}_{r} = \vec{T} \qquad \vec{T} \qquad \vec{T}_{r} = \vec{T} \qquad \vec{T} \qquad \vec{T} \times \vec{T} \qquad \vec{T} \qquad \vec{T} \qquad \vec{T} = \vec{T} \qquad \vec{T} \qquad \vec{T} \qquad \vec{T} = \vec{T} \qquad \vec{T} \qquad$$

Archimedes Law g = gravity constant We know the fluid pushes the body up with force equal to the weight of the liquid occupying the same volume as B. We take this as a fact, law of nature. Archimedes himself had a good deduction of it, nearly a provt.

Here we derive this luw using Calculus. Pascaly Law:

P = -ggZpressure grows linearly when you go down into the fluid. The force the fluid exerts on the body is inward and



Since Z is negative force decreases as you submerge the body.

This is Archimedes' Law F = (0, 0, gp Vol(B))The total force is directed vertically, and it is proportional to the mass of liquid occupying the volume of the body. Archemedes did not devive it in this way, of convise. But he came thousands of years before 1 This is the Enreka moment. Using this law, Archemodes could fell if a crown was made of pure gold: balance the crown and pure gold on a Scale in air, and then submerge it under water. It the density differs, the scale will be out of balance since the volumes will differ.

Attraction of two balls
Newton tried to build a theory of notion
of the moon around the earth.
His idea was that the force that
keeps the moon near the earth is the
same force which attracts bodies on the
surface to the center, namely gravity.
Newton's gravitational law (axiom):
Given two point masses
$$M_1$$
 and M_2
 F
 M_1 R M_2
 M_2 M_3 M_2
 M_1 R M_2
 M_1 R M_2
 M_2 M_3 M_3
 M_4 M_2
 M_1 R M_2
 M_1 R M_2
 M_2 M_3 M_3
 M_4 M_3
 M_1 M_2 M_3 M_3
 M_1 M_2 M_3 M_3
 M_1 M_2 M_3

B



Intead, we can measure slight attraction of masses











All pairs of particles attract eachother according to Newton's law. We must add up dil these contributions. The result is the attraction of the masses. This looks to be a terriby haved problem since you have to integrate over both 3P balls, making a 6-dimensional integral. This took Newton about 20 years to do... When he started, calculus did not exist so

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he had to invent it!



Consider a sphere of radius R



To find the vector equation for the force that mass m, attracts moss m by

$$\vec{F} = -Gm_{i}m_{i}\vec{r}-\vec{u}_{i}$$

$$\vec{F} = -Gm_{i}m_{i}\vec{r}-\vec{u}_{i}$$

$$\vec{r}-\vec{u}_{i}$$
Hus from m_{i} to $\vec{r}-\vec{u}_{i}$, thus from m_{i} to m_{f} .

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So, the force vector field

$$\vec{F} = \vec{F}(\vec{r})$$

has divergence zero
div $\vec{F}(\vec{r}) = 0$
as
div $\vec{F}(\vec{r}) = -Gm_{1}m \left[\frac{div(\vec{r}-u)}{||\vec{r}-\vec{u}||^{3}} - 3\frac{1}{||\vec{r}-\vec{u}||^{3}}\right] = 0$
Thus the flux over the sphere S is
A of force generated by m_{1}
 $\vec{\Phi} = \iint \vec{F} \cdot \vec{n} \, dS = -4\pi G m_{1}m$

This flux does not depend on where M, is located inside the sphere.



The Flux through any sphere containing The ball B is the same as the Flax through S.

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We now assume the following thing: the density p(i) depends only on 11 il, eig. & depends only on distance from center. Thus, if you votate the ball, the density does not change. Thus the force field F also does not change:

f is spherically symmetric !

It must be directed towards the center. The strength depends only on distance to origin

Thus, the general form of such a force is

$$\vec{F}(\vec{r}) = Cf(\vec{u}\vec{r}\vec{u})\vec{r}$$

We must find the form of \vec{q} . This
can be done using the Hax:
 $\vec{\Phi} = \int (\vec{f}(\vec{r}) \cdot \vec{n} \, ds = 4\pi R^2 \, Q(R) \cdot R$
 S
On the other hand, from before we found.
 $\vec{\Phi} = -4\pi G M m$.
Equating the two fluxes we find
 $Q(R) = -\frac{G M m}{R^3}$
 $F(\vec{r}) = -\frac{G M m}{R^3}$

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Thus the total attraction force does not change if you replace mass M, by a point mass at the center



Now we repeat this reasoning. The attraction force between m, and Mz is same as if Bz were a point mass at the center





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The dinosaurs cannot walk around inside



At best, they are flying ground.

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The End