MAT 203, Calculus III with Applications, Fall 2023

Midterm

(1) (a) (5pts) Sketch region of integration, R, whose area is given by the double integral

$$I := \iint_{R} dA = \int_{0}^{2} \int_{\frac{3}{2}x}^{3\sqrt{x/2}} dy dx.$$

- (b) (10pts) Compute the integral I.
- (c) (10pts) Change the order of integration in the integral in I.

- (2) A particle travels through three-dimensional space with acceleration $\vec{a}(t) = (0, \sqrt{2}, 2t)$, At time t = 0 the particle is at $\vec{r}(0) = (1, 0, 0)$ with velocity $\vec{v}(0) = (1, 0, 0)$.
 - (a) (5 pts) Determine the position $\vec{r}(t)$ of the particle at any time t.
 - (b) (10 pts) At time t = 0, the particle trajectory intersects the helix $\vec{H}(t) = (\cos(t), \sin(t), t)$. What is the angle between the particle's trajectory and the helix at this intersection? (HINT: how do the angles between the curves and angles between their tangent vectors relate?)
 - (c) (10 pts) How long does it take for the particle to travel 12 units of arc-length along its trajectory after passing through (1, 0, 0)?

(3) Let S be the graph of the function

$$z = 3x^2 y \sin\left(\frac{\pi}{2}x\right).$$

- (a) (10pts) Find an equation of the plane tangent to the surface S at the point where x = y = 1.
- (b) (5pts) Find the approximate value of z when x = 0.99 and y = 1.01.
- (c) (10pts) Take East to be in the direction (1,0) and North to (0,1), in which direction (i.e. North, East, South, West, Northeast, Northwest, Southeast, Southwest, Other) will a marble roll if placed on the surface S at the point above $x = 1, y = \frac{1}{2}$.

Formulas: If a surface is defined by a level set F(x, y, z) = 0, the tangent plane at a point $\vec{r}_0 := (x_0, y_0, z_0)$ on the surface is the collection of points $\vec{r} = (x, y, z)$ so that

$$\nabla F(\vec{r}_0) \cdot (\vec{r} - \vec{r}_0) = 0.$$

The normal line through point $\vec{r_0}$ is the collection of points \vec{r} such that

$$\vec{r} = \vec{r}_0 + t\nabla F(\vec{r}_0)$$

for any number t.