MAT 203, Calculus III with Applications, Fall 2023 — Homework 9

M&T Sections: 5.3, 6.2, 8.1

(1) Find the volume of the region boundary by $x^2 + y^2 = 1$, $x = z$ and $z = 0$. This region is known as the *hoof of Archimedes*.

(2) Evaluate the following integrals using polar coordinates

- $I = \int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dxdy$,
- $I = \int_{\sqrt{2}}^2 \int_{\sqrt{4-y^2}}^y dxdy$,
- $I = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2} dydx$.

(3) Use Green's theorem to compute $\oint_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = (\arctan(y/x), \log(x^2 + y^2))$ along the curve C given by the boundary of the region defined by the polar inequalities $1 \leq r \leq 2$ and $0 \leq \theta \leq \pi/2$, oriented counterclockwise.

(4) Find the work done by the force $\vec{F} = (4x - 2y, 2x - 4y)$ on a particle going counterclockwise around the circle C: $(x - 2)^2 + (y - 2)^2 = 4$.

(5) Show that the value of $\oint_C xy^2dx + (x^2y + 2x)dy$ around any square C depends only on the area enclosed and not on its location in the plane.