(1) Find the tangent plane to the the surface defined by $f(x, y) = x^3 - xy + y^2$ at $(2, 1)$.

(2) Find the unit normal to the *graph* (namely, the two-dimensional surface sitting in three-dimensional space defined by $z = f(x, y)$) for $f(x, y) = e^x y$ at the point $(-1, 1)$.

(3) Captain Buzz is in trouble near the sunny side of Mars, at coordinate $(1, 1, 1)$. His ship's hull is melting. He measures the temperature in his vicinity to be $T(x, y, z) = e^{-x} + e^{-2y} + e^{3z}$. In what direction should he proceed in order to cool the fastest?

(4) Compute:
- f_{xyzxz} if $f(x, y, z) = x \exp(yz^2 \sin(y + z))$,
- $f_{tt} - f_{xx}$ if $f(x, t) = \sin(x) \cos(t)$,
- all second order partial derivatives if $f(x, y) = x \log y$.

(5) Describe what div$\vec{v}(\vec{r})$ and curl$\vec{v}(\vec{r})$ means and compute them for the vector fields:

- $\vec{v}(\vec{r}) = \|\vec{r}\|\vec{r}$ where $\vec{r} = (x, y, z)$,

- $\vec{v}(r) = \frac{\vec{r}}{\|\vec{r}\|^3}$ where $\vec{r} = (x, y, z)$,

- $\vec{v}(x, y, z) = (A\sin z + C\cos y, B\sin x + A\cos z, C\sin y + B\cos x)$,

(6) Let ∇^2 be the Laplacian operator defined by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

- If f is a twice-differentiable function, show that $\mathrm{div}(f\nabla f) = \|\nabla f\|^2 + f\nabla^2 f$
- Suppose $\vec{F}$ is a C^2 vector field, show that $\mathrm{curl}(\mathrm{curl}\vec{F}) = \nabla(\mathrm{div}\vec{F}) - \nabla^2\vec{F}$

(7) Consider the function $f(x, y) = \ln(\sqrt{x^2 + y^2} + y)$.

- Determine the domain of f and sketch it in the xy-plane.

- What is the linearization of f at $(3, -4)$?