(1) (M&T, # 2.1.10) Describe the behavior, as c varies, of the level curves $f(x,y)=c$ for each of these functions

- $f(x,y)=x^2+y^2+1$,
- $f(x,y)=1-x^2-y^2$,
- $f(x,y)=x^3-x$.

(a) The first function has levels which are circles, for any $c>1$. For $c=1$ it is a point. For $c<0$, it is empty.
(b) Levels are likewise circles, but now for $c<1$. For $c=1$ it is a point and for $c>1$ it is empty.
(c) Level sets are sheets for any real c, function curves $x^3-x=c$, extruded in the y direction.

(2) (M&T, # 2.3.1) Find $\partial f/\partial x$, $\partial f/\partial y$ if

- $f(x,y)=xy$
- $f(x,y)=e^{xy}$
- $f(x,y)=x\cos x\cos y$
- $f(x,y)=(x^2+y^2)\log(x^2+y^2)$.

(a) $\partial_x f=y$, $\partial_y f=x$
(b) $\partial_x f=ye^{xy}$, $\partial_y f=xe^{xy}$
(c) $\partial_x f=\cos x\cos y-x\sin x\cos y$, $\partial_y f=-x\cos x\sin y$
(d) $\partial_x f=2x\log(x^2+y^2)+2x$, $\partial_y f=2y\log(x^2+y^2)+2y$

(3) (M&T, # 2.5.10) Suppose that the temperature at the point (x,y,z) in space is $T(x,y,z)=x^2+y^2+z^2$. Let a particle follow the helix $\vec{r}(t)=(\cos t,\sin t,t)$ and let $T(t)$ be its temperature at time t.

- What is $T'(t)$?
- Find an approximate value for the temperature at $t=\frac{\pi}{2}+0.01$.

$T(t)=\cos^2 t+\sin^2 t+t^2$. Thus $T'(t)=-\sin t\cos t+\cos t\sin t+2t=2t$. The approximation is $T(\frac{\pi}{2}+0.01)\approx T(\frac{\pi}{2})+(0.01)\cdot T'(\frac{\pi}{2})=1+(\pi/2)^2+0.01\pi$.

(4) (M&T, # 2.5.18, modified) Compute the directional derivative of f in the given directions $\vec{v}$ at the given points P.

- $f(x,y,z)=xy^2+y^2z^3+z^3x$, $P=(4,-2,1)$, $\vec{v}=\frac{1}{\sqrt{14}}(\vec{i}+3\vec{j}+2\vec{k})$
- $f(x,y,z)=e^{-z}\sin(x)\sin(y)$, $P=(\pi,\frac{\pi}{2},0)$, $\vec{v}=\frac{12}{13}\vec{i}+\frac{3}{13}\vec{j}+\frac{4}{13}\vec{k}$.

$\nabla f=(y^2+z^3,2yx+2yz^3,3z^2y^2+3z^2x)$. Thus $\nabla f|_P=(5,-20,24)$. $D_{\vec{v}}f|_P=-\sqrt{\frac{7}{2}}$.
$\nabla f=e^{-z}(\cos x\sin y,\sin x\cos y,\sin x\sin y)$. Thus $\nabla f|_P=(-1,0,0)$. $D_{\vec{v}}f|_P=-\frac{12}{13}$.

(5) (M&T, # 2.5.28) In electrostatics, the force $\vec{P}$ of attraction between two particles of opposite charge is given by $\vec{P}=k\frac{\vec{r}}{\|\vec{r}\|^3}$ (Coulomb's law), where k is a constant and $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$. Show that $\vec{P}$ is the gradient of $f=-\frac{k}{\|\vec{r}\|}$.

Note that $\nabla\|\vec{r}\|=\frac{\vec{r}}{\|\vec{r}\|}$. Thus $\nabla\|\vec{r}\|^{-1}=-\frac{1}{\|\vec{r}\|^2}\nabla\|\vec{r}\|=-\frac{\vec{r}}{\|\vec{r}\|^3}$. The result follows.