(1) (M&T, # 4.1.26) Let $\vec{r}(t)$ be the vector from the origin to the position of an object of mass $m > 0$, $\vec{v}(t) := \vec{r}'(t)$ be the velocity and $\vec{a}(t) := \vec{r}''(t)$ be the acceleration. Suppose that $\vec{F}(t)$ is the force acting at time t, so that $\vec{F} = m\vec{a}$.

(a) Prove that $\frac{d}{dt}(m\vec{r} \times \vec{v}) = \vec{r} \times \vec{F}$. What do you conclude if $\vec{F}$ is parallel to $\vec{r}$?

(b) Prove that a planet (say, of mass m) moving about the Sun (say, of mass M) does so in a fixed plane (one of Kepler's Laws). Recall, for planetary motion, $\vec{F} = -\frac{GmM}{\|\vec{r}\|^3}\vec{r}$.

(2) Let $\vec{r}(t) = (\cos(t), \sin(t), t)$ be the helix.

(a) Find the length of the helix for $0 \le t \le \pi$.

(b) Find the arc length parametrization for the helix.

(3) Compute the curvature and principle normal vector of the helix $\vec{r}(t) = (\cos(t), \sin(t), t)$.

(4) Compute the curvature of the exponential spiral $\vec{r}(t) = (e^t \cos(t), e^t \sin(t), 0)$. Draw a (rough) picture of this curve. What happens as $t \to \infty$?

(5) Prove that if the curvature of a curve is identically zero, then the curve is a straight line.