(1) (M& T, # 4.1.12) The acceleration of a particle is $\vec{r}''(t) = (0, 0, 6)$. Its initial velocity is $\vec{r}'(0) = (1, 1, -2)$ and initial position is $\vec{r}(0) = (3, 4, 0)$. Find $\vec{r}'(t)$ and $\vec{r}(t)$.
$\vec{r}'(t) = (0, 0, 6)t + (1, 1, -2)$. $\vec{r}(t) = (0, 0, 3)t^2 + (1, 1, -2)t + (3, 4, 0)$.

(2) (M& T, # 4.1.14) The acceleration of a particle is $\vec{r}''(t) = (-6, 2, 4)$. Its initial velocity is $\vec{r}'(0) = (2, -5, 1)$ and initial position is $\vec{r}(0) = (-3, 6, 2)$. The particle's trajectory intersects the xz plane exactly twice. Find these two intersection points.
$\vec{r}'(t) = (-6, 2, 4)t + (2, -5, 1)$. $\vec{r}(t) = (-3, 1, 2)t^2 + (2, -5, 1)t + (-3, 6, 2)$. We have $0 = y(t) = t^2 - 5t + 6$. The two roots are $t = 2, 3$. The points are found by plugging in.

(3) (M& T, # 4.1.20) Let $\vec{r}(t)$ be a differentiable vector valued function of t. Show that, at a local maximum or minimum of $\|\vec{r}(t)\|$, the vector $\vec{r}'(t)$ is perpendicular to $\vec{r}(t)$.
$0 = \frac{1}{2}\frac{d}{dt}\|\vec{r}(t)\|^2 = \vec{r}'(t) \cdot \vec{r}(t)$.

(4) Consider a particle with position $\vec{r}(t)$ given by

$$\vec{r}(t) = (a \cos \omega t, a \sin \omega t, b\omega t), \qquad a \neq 0.$$

(a) Show the speed of the particle is constant.
$\vec{r}'(t) = \vec{v}(t) = (-a\omega \sin \omega t, a\omega \cos \omega t, b\omega)$, $\|\vec{r}'\| = \sqrt{a^2 + b^2}\omega$.

(b) Show that the acceleration vector is always parallel to the xy plane.
$\vec{r}''(t) = \vec{a}(t) = (-a^2\omega \cos \omega t, -a^2\omega \sin \omega t, 0)$, $\vec{r}'' \cdot \hat{k} = 0$.

(c) Show the velocity of the particle makes a constant non-zero angle with the z-axis.
$\vec{v} \cdot \hat{k} = b\omega$ is constant so the angle $\cos\theta = \frac{b\omega}{\sqrt{a^2+b^2}\omega} = \frac{b}{\sqrt{a^2+b^2}}$ is constant.

(d) Note $P = \vec{r}(0) = (a, 0, 0)$ and $Q = \vec{r}(\frac{2\pi}{\omega}) = (a, 0, 2\pi b)$. So $\overrightarrow{PQ}$ is vertical. Show that

$$\vec{r}(\frac{2\pi}{\omega}) - \vec{r}(0) = \frac{2\pi}{\omega}\vec{r}'(s)$$

cannot hold for any $s \in (0, \frac{2\pi}{\omega})$. Therefore, the Mean Value Theorem does not hold for vector valued functions.
Since $\overrightarrow{PQ}$ is vertical, and $\vec{v}$ is never vertical, $\overrightarrow{PQ}$ can not be $\frac{2\pi}{\omega}\vec{v}(s)$ for any s.