(1) (M&T, # 4.1.12) The acceleration of a particle is $\vec{r}''(t) = (0, 0, 6)$. Its initial velocity is $\vec{r}'(0) = (1, 1, -2)$ and initial position is $\vec{r}(0) = (3, 4, 0)$. Find $\vec{r}'(t)$ and $\vec{r}(t)$.

(2) (M&T, # 4.1.14) The acceleration of a particle is $\vec{r}''(t) = (-6, 2, 4)$. Its initial velocity is $\vec{r}'(0) = (2, -5, 1)$ and initial position is $\vec{r}(0) = (-3, 6, 2)$. The particle's trajectory intersects the xz plane exactly twice. Find these two intersection points.

(3) (M&T, # 4.1.20) Let $\vec{r}(t)$ be a differentiable vector valued function of t. Show that, at a local maximum or minimum of $\|\vec{r}(t)\|$, the vector $\vec{r}'(t)$ is perpendicular to $\vec{r}(t)$.

(4) Consider a particle with position $\vec{r}(t)$ given by

$$\vec{r}(t) = (a\cos\omega t, a\sin\omega t, b\omega t), \qquad a \neq 0.$$

(a) Show the speed of the particle is constant.

(b) Show that the acceleration vector is always parallel to the xy plane.

(c) Show the velocity of the particle makes a constant non-zero angle with the z-axis.

(d) Note $P = \vec{r}(0) = (a, 0, 0)$ and $Q = \vec{r}(\frac{2\pi}{\omega}) = (a, 0, 2\pi b)$. So $\overrightarrow{PQ}$ is vertical. Show that

$$\vec{r}(\frac{2\pi}{\omega}) - \vec{r}(0) = \frac{2\pi}{\omega}\vec{r}'(s)$$

cannot hold for any $s \in (0, \frac{2\pi}{\omega})$. Therefore, the Mean Value Theorem does not hold for vector valued functions.