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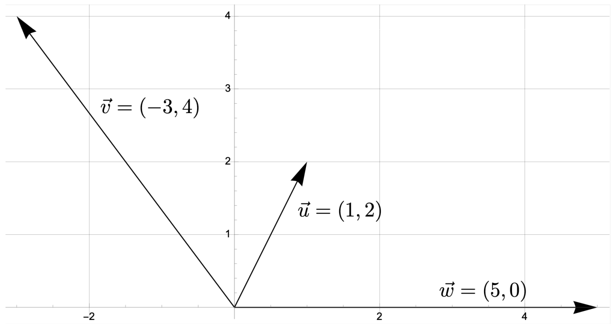
(1) (M&T, §1.1: #22) Let $\vec{u} = (1, 2)$, $\vec{v} = (-3, 4)$, and $\vec{w} = (5, 0)$.
- Draw these vectors in $\mathbb{R}^2$.
- Find scalars λ_1 and λ_2 so that $\vec{w} = \lambda_1\vec{u} + \lambda_2\vec{v}$.



We have $\lambda_1\vec{u} + \lambda_2\vec{v} = (\lambda_1 - 3\lambda_2, 2\lambda_1 + 4\lambda_2) = (5, 0)$. Thus $\lambda_1 = -2\lambda_2$ and $\lambda_1 - 3\lambda_2 = -5\lambda_2 = 5$, so $\lambda_2 = -1$ and $\lambda_1 = 2$.

(2) (M&T, §1.2: #15) What is the geometric relation between the vectors $\vec{u}$ and $\vec{v}$ if one has that $\vec{u} \cdot \vec{v} = -\|\vec{u}\|\|\vec{v}\|$? What is a formula for $\text{Proj}_{\vec{u}}\vec{v}$?

The two vectors are antiparallel. We have $\text{Proj}_{\vec{u}}\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\vec{u} = -\frac{\|\vec{v}\|}{\|\vec{u}\|}\vec{u}$.

(3) Suppose A, B and C are collinear, and

$$\frac{\|\overrightarrow{AB}\|}{\|\overrightarrow{BC}\|} = \frac{\lambda}{1-\lambda}, \qquad \lambda \in (0, 1).$$

Show that $B = (1 - \lambda)A + \lambda C$.

From the above, one can readily see:

$$\overrightarrow{AB} = \lambda\overrightarrow{AC}, \qquad B - A = \lambda(C - A).$$

(4)
- (M&T, §1.2: #6) Compute $\|\vec{u}\|$, $\|\vec{v}\|$, $\vec{u} \cdot \vec{v}$ for $\vec{u} = 15\vec{i} - 2\vec{j} + 5\vec{k}$ and $\vec{v} = \pi\vec{i} + 3\vec{j} - \vec{k}$. $\vec{u} = (15, -2, 5)$ thus $\|\vec{u}\|^2 = (15)^2 + 2^2 + 5^2 = 254$ and $\vec{v} = (\pi, 3, -1)$ thus $\|\vec{v}\|^2 = (\pi)^2 + 3^2 + 1 = \pi^2 + 10$. Finally, $\vec{u} \cdot \vec{v} = 15\pi + 3 \times (-2) + 5 \times (-1) = 15\pi - 11$.
- (M&T, §1.2: #20) Find the projection of $\vec{u} = -\vec{i} + \vec{j} + \vec{k}$ onto $\vec{v} = 2\vec{i} + \vec{j} - 3\vec{k}$.

$$\text{Proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\vec{v} = -\frac{4}{3}(2, 1, -3).$$

(5)
- Find the length of $\vec{v} = 2\vec{i} - 6\vec{j} + 7\vec{k}$. $\vec{v} = (2, -6, 7)$ thus $\|\vec{v}\|^2 = 2^2 + 6^2 + 7^2 = 4 + 36 + 49 = 89$. Thus, $\|\vec{v}\| = \sqrt{89}$.
- Find all the values of c for which $\|\vec{i} + \vec{j} + c\vec{k}\| = 4$. Note $\vec{i} + \vec{j} + c\vec{k} = (1, 1, c)$ so $\|\vec{i} + \vec{j} + c\vec{k}\|^2 = 1 + 1 + c^2 = 2 + c^2$. Thus we require $2 + c^2 = 16$ or $c = \pm\sqrt{14}$.