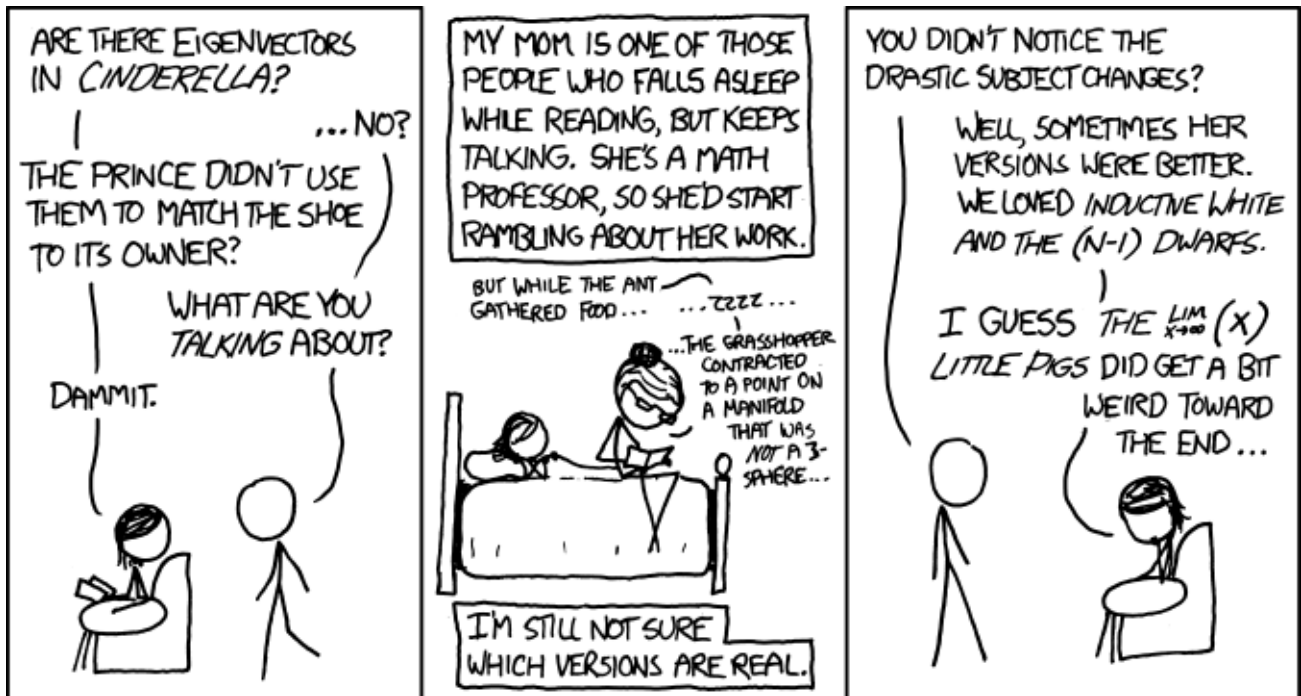


The $\lim_{x \rightarrow \infty} x$ Little Pigs

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Randall Munroe, the creator of the popular webcomic xkcd has this comic about fairy tales and math mixing together. This is quite an old webcomic, number 872. However, when I looked to see if anyone has actually written a version of the fairy tale, I didn't find any. Of course, I didn't look that hard; I was already thinking of my own version of the story.



Thus, I decided to try writing a few of my own spins on classic fairy tales. I tried to stick somewhat close to the original stories. This is surely contestable because in the case of Cinderella, say, there are many versions, including the Brothers Grimm and also Disney. I would lean towards the Disney version both because it's more well-known and also, less gruesome. I also wanted to stick to Munroe's original idea. I'm not sure how to incorporate eigenvectors into Cinderella but there you go.

Here, I'm trying my hand at the tale "The Three Little Pigs." The original story goes something like this. Once upon a time, there were three little pigs who were siblings but each lived in his or her own house. The first pig, which I'll creatively call A, lived in a house of straw. One day, a big bad wolf came and saw the pig and also that the house was made of straw. So he huffed and he puffed and he blew the house down, in order to devour A. A ran away to B's house which is made of sticks. The wolf follows, blows that one down and so A and B run to C's house. C's house is made of bricks and the wolf is unable to blow it down. The end.

Apparently, there are versions where the wolf eats A and B but can't eat C. There are also versions where the wolf climbs the chimney of C's house and falls into a cauldron of boiling water in which the pigs cook and then eat the wolf. Gruesome. And I can't tell what the moral of the story is except perhaps to make wiser building material choices.

1 The Story

Once upon a time, there were a lot of pigs. There were, in fact, as many pigs as there are counting numbers. Setting aside the fact that their mother must have been exhausted birthing and raising all these pigs, there was also the issue of naming them all. So as you might expect, the pigs were named Eins, Zwei, Drei, Vier, Fünf, and so on.

One day, when the pigs were all adults, they decided to all move to the same neighborhood and live in their own separate homes. Each pig did not have much of an inheritance since, well, dividing a finite amount of money between $|N|$ pigs doesn't amount to much. So each pig had to make do with what they could find to build their house. Basically, all of them made their houses out of straw.

After awhile, the abundance of pigs caught the attention of a big, bad, wolf. The wolf saw the house of Eins, and decided that he had enough of the few scraps a day. He wanted ham. So he went up to the door of the first house and said:

“Little pig, little pig, let me come in.”

Eins was no fool. He knew, as does the narrator of this story, that this wolf is big and bad. I mean, this wolf never had a chance to be anything but big and bad. Poor wolf. So Eins said to the wolf:

“No, no, by the hair on my chinny chin chin.”

The wolf pondered this for a moment. Who on earth would swear by the hair on their chinny chin chin? No matter.

“Then I'll huff, and I'll puff, and I'll blow your house in.”

And that's what happened. The big, bad wolf huffed and puffed and blew the house in. In the collapse of the house, Eins ran off towards Zwei's house. The wolf was after Eins in hot pursuit but Eins made it to Zwei's house before he was caught.

The illogical wolf, not seeing that he could simply resort to bursting through the straw facade of false security, went ahead and called to the pigs:

“Little pigs, little pigs, let me come in.”

Well, the whole thing repeated and you, dear reader, must know by now how the story goes. But we break the golden rule of three. There are not three pigs. Indeed, the wolf discovers that there seems to be an unending sequence of pigs. And as such, this story has no end. Each of the first n pigs run to the house of the $n + 1$ st pig. There is no end.

2 A Version Without ∞

For those who don't like that there are an infinite number of pigs or would just like there to be an end to the story or are just extreme realists that reject the notion of infinity, fine. Let's just say that there are lots of pigs, but finitely many. Bigger than any number you care to name but still finite. Okay. The story goes as before but...

When around 2×10^{28} pigs have gathered together at the next pigs house, there's enough mass to form a blackhole and so the earth and all its inhabitants are drawn into the abyss. In the end, all the pigs and the wolf are eaten by the blackhole and the universe persisted onwards in its inevitable march towards entropic death. The end.

3 The Morals of the (1st) Story

- If you have an engineering problem, such as only having straw to build a house to keep out a wolf, sometimes, you can just ignore it by pushing the problem to infinity. There is no “last straw house” and so there is no end to the chase. It's what topologists often do, like Freedman performing the Whitney trick on 4-manifolds *ad infinitum* or the construction of Postnikov towers.
- This story illustrates the simplest version of Hilbert's hotel, somewhat.