Homework 13

5.3

1,4,7,8 see pp. 313-314

2. Consider the following diagram:

$$\begin{array}{ccc} 2 & ------ & 3 \\ | & & | \\ 1 & ------ & 4 \end{array}$$

Acting by reflection σ, we get

$$\begin{array}{ccc} 3 & ------ & 2 \\ | & & | \\ 4 & ------ & 1 \end{array}$$

acting by rotation R, we get

$$\begin{array}{ccc} 4 & ------ & 1 \\ | & & | \\ 3 & ------ & 2 \end{array}$$

So the corresponding permutations are $\sigma = (14)(23), R = (13)(24)$. This is an isomorphism different from the book. We can index the vertices of the square differently, and then check the isomorphism that we get similarly as above. From here we can see that there are six isomorphisms.

9. Since elements with order 2 in $A(4)$ are $(12)(34), (13)(24), (14)(23)$, and it is easy to check that multiple two of them together we will get the one left for example $(12)(34)(13)(24) = (14)(23)$. Thus for any two order 2 elements σ, γ in $A(4)$ where $\sigma \neq \gamma$, $\sigma\gamma$ has order 2. Assume that $H \subset A(4)$ is a subgroup of 6 elements. Since $A(4)$ does not have element of

order 6, thus H is not cyclic. If H is isomorphic to $S(3)$, in $S(3)$, $(12), (13)$ have order 2, but $(12)(13) = (132)$ which has order 3 this contradicts to our observation of order 2 elements in $A(4)$. So $A(4)$ has no subgroup of order 6.

5.4

2,4,5 see p. 314