

Homework 12

5.1

6,10 - see pp. 311-312

5.2

1,3,5 - see pp. 312-313

2. Let n be any integer greater than 3. Let $k = n - 1$. n and k satisfy the linear diophantine equation $(1)n + (-1)k = 1$, so the greatest common divisor of n and k is 1, in other words n and k are relatively prime. Thus, by 5.2.7, $k^{\phi(n)} \equiv 1 \pmod{n}$. But $k \equiv (-1) \pmod{n}$, so we have $(-1)^{\phi(n)} \equiv 1 \pmod{n}$. Now, suppose $\phi(n)$ were odd. Then we would have $\phi(n) = 2m + 1$ for some m . So $(-1)^{\phi(n)} \equiv (-1)^{2m+1} \equiv (-1) \pmod{n}$, which gives us $1 \equiv -1 \pmod{n}$. But $n \geq 3$, so $1 \not\equiv -1 \pmod{n}$. Thus, $\phi(n)$ must be even.