SKETCH OF SOLUTIONS (HOMEWORK II)

4.- a) yes. b)no, c) yes, d) yes, e) yes, f) no

16.- Suppose the division algorithm yields $a = qb + r$, $q = ct + s$ with $r < b$, $s < c$. Then substituting we get:

$$a = (cb)t + (sb + r)$$

If we show that $sb + r < cb$ we are done (*why?*). Notice $s < c \Rightarrow s + 1 \leq c$ Thus $(s + 1)b \leq cb$ but since $r < b$ we get $sb + r < sb + b \leq cb$

28.- We must require $m, n > 0$

$$\left[\frac{x+n}{m}\right] = \left[\frac{[x+n]}{m}\right] = \left[\frac{[x]+n}{m}\right]$$

The first equality comes from example 1.34 and the second from example 1.31

Section 3.1

2.- a) not prime, b) not prime, c) not pime, d) prime, e) not prime, f) not prime

6.- Notice $n^3+1 = (n+1)(n^2-n+1)$ Therefore n^3+1 is prime iff $n+1 = n^3+1$ (and $n + 1 > 1$) iff $n = 1$

10.- Let p_i be a prime in the list. Without loss of generality we can say p_i divides Q and p_i does not divide R. But then p_i cannot divide $Q + R$. Since $Q + R > p_n$ there must be more than n primes.

Section 3.2

6.- If $b|a$ and $b|a + 2$ then $b|a + 2 - a$ therefore $\gcd(a, a + 2)$ is either 1 or 2. If a is even then $\gcd(a, a + 2) = 2$

7.- By theorem 3.8 we kno (ca, cb) is the least positive integer of the form $cma + cmb = |c||ma + nb|$ therefore $|ma + nb|$ is minimum i.e. $|ma + nb| = (ma, nb)$

31.- Suppose $\frac{ad+bc}{bd} = k$ then $kbd = ad + bc$ therefore $d|bc$. But $d \nmid c$, therefore we must have $d \mid b$. Analogously $b \mid d$