**SKETCH OF SOLUTIONS (HOMEWORK VII)**

19.1- (a) $\lambda(30) = -1$, $\lambda(504) = 1$, $\lambda(60750) = -1$

(b)

$$G(n) = \begin{cases} 0 & \text{if} \quad n = 1,2,3,5,6,7,8,10,11,12,13,14,15,17,18 \\ 1 & \text{if} \quad n = 4,9,16 \end{cases}$$

(c)

$$G(n) = \begin{cases} 0 & \text{if} \quad n \text{ is not a perfect square} \\ 1 & \text{if} \quad n \text{ is a perfect square} \end{cases}$$

(d) $\lambda$ is multiplicative, therefore $G$ is multiplicative. $G(p^k) = 0$ if $k$ is odd and $G(p^k) = 1$ if $k$ is even.

19.3- (a) $\sigma_2(12) = 210$, $\sigma_3(10) = 1134$, $\sigma_0(18) = 6$

(b) Same proof as the proof for the claim on page 122. It is not true if $\gcd(m,n) \neq 1$ take $m = n = 2$ as a counterexample

(c)

$$\sigma_t(p^k) = \begin{cases} \frac{p^{t(k+1)}-1}{p^t-1} & \text{if} \quad t > 0 \\ 1+k & \text{if} \quad t = 0 \end{cases}$$

$\sigma_4(2^6) = 17895697$

(d) Yes. $\sigma_0(42336000) = 432$

- $\sigma(2^{100}) = 2^{101} - 1$, $\sigma(196) = 399$, $\sigma(20!) = 13891399238731734720$
- Numbers which are squares.
- $n = 4$
- Either $n = pq$ with $p, q$ primes or $n = p^3$ with $p$ prime.
- Hint: Prove the following formula
$$\Pi_{d|n} d = n^{\frac{\tau(n)}{2}}$$
- $\mu(12) = 0$, $\mu(30) = -1$
- $4 \mid n(n+1)(n+2)(n+3)$
- Notice that the linear system of congruences
$$\begin{aligned} x &\equiv 1 \mod 4 \\ x &\equiv 2 \mod 9 \\ x &\equiv 3 \mod 25 \\ x &\equiv 4 \mod 49 \\ x &\equiv 5 \mod 121 \end{aligned}$$
has a solution $x$ (*why?*) such that $\mu(x-1) = \mu(x-2) = \mu(x-3) = \mu(x-4) = \mu(x-5) = 0$
- $$\phi(n) = n \sum_{d|n} \frac{\mu(d)}{d}$$

13.3 Notice that if $1 < k \leq n$, $\frac{n!}{k}$ is an integer and $n! + k = k(\frac{n!}{k} + 1)$ is composite