MAT 126 **Practice Midterm I** **October 13, 2003**

Name: _____ **Id:** _____

You have 90 minutes to complete the midterm exam. This practice exam is perhaps longer than a real exam. You may use calculators, provided that they do not do any symbolic manipulations (e.g. Ti83 and the likes). You may **NOT** use any books or notes. Please write full solutions, not just answers. **Show your work**, **explain your reasoning**, and cross out anything we should ignore when grading. Where possible, give exact answers (for example, "$\sqrt{5}$" rather than "2.23").

This practice midterm exam has 6 questions, for a total of 125 points. Good luck!

1. Compute the following antiderivatives. Show all your work!
   (a) (10 points) $\displaystyle\int \frac{x^2+1}{\sqrt{x}}\,dx, \qquad x \in (0,\infty)$
   (b) (10 points) $\displaystyle\int \frac{1}{x^2+1}\,dx$
   (c) (10 points) $\displaystyle\int \frac{1-x}{1+\sqrt{x}}dx, \qquad x \in (0,\infty)$

2. A car travels Eastward with the velocity $V(t) = 3t^2 - 2$ (in miles per hours) at time t (in hours).
   (a) (10 points) What is the displacement after three hours?
   (b) (10 points) What is the distance traveled in the same interval of time?

3. (a) (10 points) Compute L_4 and R_4 for the integral
   $$\int_0^4 \frac{1}{x^2+1}\,dx.$$
   (**Hint:** In both cases you have to use 4 equidistant subintervals, but the heights of the rectangles are found with left-hand endpoints in the case of L_4 and with right-hand endpoints in the case of R_4)
   (b) (10 points) Is L_4 an underestimate or an overestimate for the exact value of the integral? Explain.

4. (20 points) Let $I = \int_0^3 f(x)\,dx$ for some function f. If $f(x) = 1 + \sqrt{9-x^2}$ compute I by interpreting it in terms of areas.

5. A metal rod of length $\frac{\pi}{6}$ meters has a weight distribution of $\rho(x) = \cos x$ kg/meter, where x is the distance in meters from one of the ends.
   (a) (10 points) Find the weight of the rod.
   (b) (10 points) Find where to cut the rod in two so that both pieces have the same weight.

6. (a) (5 points) Find the derivative $f'(x)$ of the function f defined as follows

$$f(x) = \int_0^{x^2} \sqrt{1+t^2}\, dt, \qquad x \in \mathbb{R}$$

(b) (5 points) Find the intervals on which the above function f is increasing or decreasing

(c) (5 points) Find the intervals on which the above function f is concave up or concave down.