

MAT 536

Midterm

March 22, 2023

Name: _____ ID: _____

Question:	1	2	3	4	Total
Points:	10	10	10	10	40
Score:					

There are 4 problems on 4 pages (plus this cover sheet) in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted. No electronic devices may be used AT ALL.** If you can channel the spirit of Euler, Riemann, Weierstrass, or Cauchy, feel free to use your psychic connection for help, but according to the academic integrity policy, you must cite them appropriately. Don't ask Lars Ahlfors for help, he's very tired today.

You have 90 minutes to complete this exam.

10 pts

1. Let $U \subset \mathbb{C}$ be a simply connected domain, and let f be meromorphic on U . Suppose that for each pole p of f , the residue of f at p is zero. Show that f has a primitive F on U .

10 pts

2. Let $f(z) = z^9 + z^5 - 9z^3 + 3z + i/3$.

How many zeros does f have in the annulus $1/3 < |z| < 1$? Fully justify your answer.

10 pts 3. Let f be a non-constant entire function. Show the that $f(\mathbb{C})$ is dense in \mathbb{C} .

- 10 pts 4. Let f be meromorphic in a simply connected domain U , and let p be a pole of f . Show that no pole p of f is also a pole of $e^f = \exp \circ f$.