1. Show that

$$f(z) = \sum_{n=1}^{\infty} \left(1 - \cos\left(\frac{z}{n}\right)\right)$$

is an entire function. (Hint: Prove the estimate $|1 - \cos w| \le C|w|^2$ for $|w| < 1$ and use the Weierstrass M-test.)

2. Let r_n denote the distance between 0 and the closest root of the polynomial

$$f_n(z) = 1 + z + \frac{z^2}{2!} + \cdots + \frac{z^n}{n!}.$$

Show that $\lim_{n\to\infty} r_n = +\infty$. (Use Hurwitz's theorem).

3. Let $\mathcal{F} \subset \mathcal{O}(U)$.

   (a) Show that if $\mathcal{F}$ is precompact, then so is the family of kth-derivatives $\mathcal{F}^{(k)} = \{f^{(k)} \mid f \in \mathcal{F}\}$.

   (b) Show by example that $\mathcal{F}'$ precompact does not imply that $\mathcal{F}$ is precompact.

4. **(a)** Show that the family of holomorphic functions $f : \mathbb{D} \to \mathbb{D}$ is precompact.

   (b) Let $f_n \in \mathrm{Aut}(\mathbb{D})$ with $f_n(z) = \dfrac{z - p_n}{1 - \overline{p_n}z}$. Describe the limit function $f(z)$ as $p_n \to p$ with $p \in \mathbb{D}$ and the limit when $|p| = 1$.

5. Prove that the family of all $f \in \mathcal{O}(\mathbb{D})$ with $\mathrm{Re}(f) > 0$ is normal (that is, it is precompact in $\mathcal{C}(\mathbb{D}, \widehat{\mathbb{C}})$), but it is not precompact in $\mathcal{C}(\mathbb{D}, \mathbb{C})$.