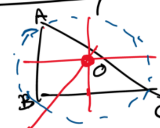
TriangleCenters

Wednesday, October 21, 2020 5:59 PM

DEF: 3 OR MORE LINES MEETING AT A POINT ARE CONCURRENT

HAVE ALREADY SEEN THE THREE PERP. BISECTORS OF THE SIDES OF A TRIANGLE ARE CONCURRENT O AT THE CIRCUMCENTER OF ABC

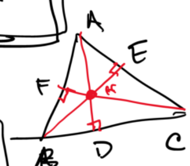


COULD BE OUT

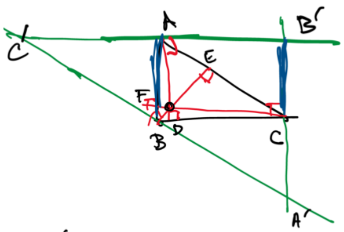
OTHER SUCH POINTS (CALLED CENTERS)

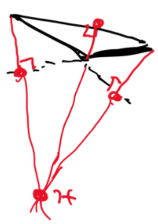
THM THREE ALTITUDES OF A TRIANGLE ARE CONCURRENT AT THE ORTHOCENTER H

Pf/ LET ALTITUDES BE $\overleftrightarrow{AD}$, $\overleftrightarrow{BE}$, $\overleftrightarrow{CF}$



[NOTE IF ACUTE, $D \in \overline{BC}$, $E \in \overline{AC}$, $F \in \overline{AB}$.
OBTUSE, TWO OUTSIDE]





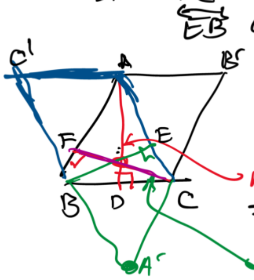
Pf/ GIVEN $\triangle$ A, B, C, DRAW PARALLELS TO OPP SIDES THRU EACH VERTEX.

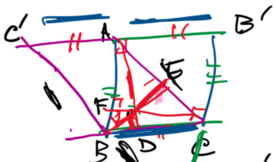
A = MIDPOINT OF B'C'
B = MIDPOINT OF A'C'

$\square AB'CB$ IS //OGRAM
C = MIDP. OF B'A'

$\overleftrightarrow{AD} \perp \overleftrightarrow{BC}$, $\overleftrightarrow{BC} \,//\, \overleftrightarrow{C'B'}$, $\Rightarrow \overleftrightarrow{AD} \perp \overleftrightarrow{B'C'}$

SAME $\overleftrightarrow{FC}$
$\overleftrightarrow{EB}$





ALT(AB') = ALT(AC')

ALT (ACA'B) & AB'BC

//A'

COMPARE ALL //OGRAMS, SEE THE MEET IN SAME WAY.

AD IS ALT. OF $\square$ AB'CB & BC'ACB

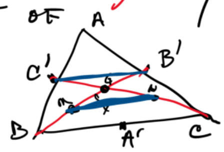
FC IS OTHER ALT OF $\square$ AB'CB & ACA'B.

THIS PROOF IS DUE TO GAUSS.
LOTS OF OTHERS, BUT LONGER.
THIS IS "CLEVER".

THM: THE THREE MEDIANS ARE CONCURRENT AT CENTROID (G).
(ii) FURTHER, DISTANCE FROM G TO A VERTEX IS TWICE THE DIST. FROM G TO THE MIDPOINT OF OPP. SIDE.

KEY TO PROOF

$|AG| = 2|GA'|$



Pf/ LET A' = MID(BC) C' = MID(AB)
B' = MID(AC)

G = INTERSECTION OF $\overline{CC'}$ WITH BB'

X = POINT ON BB' WITH $|BX| = 2|B'X|$

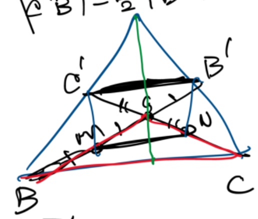
NEED TO SHOW THAT $G = X$, IE
X ON BOTH $\overline{CC'}$ AND BB'

CONNECT C' & B'

$\overleftrightarrow{C'B'} \,//\, \overleftrightarrow{BC}$ BY FTS (CUZ $\frac{|AC'|}{|AB|}$) $= \frac{|AB'|}{|AC|} = \frac{1}{2}$

$|C'B'| = \frac{1}{2}|BC|$

M = MIDPOINT $\overline{GB}$
N = MIDPOINT OF GC



$\overleftrightarrow{MN} \,//\, \overleftrightarrow{BC}$ BY FTS
$|MN| = \frac{1}{2}|BC|$

SO $\overline{C'B'} \,//\, \overline{MN}$, SAME LENGTH

SO $\square$ MNB'C' IS A //OGRAM.

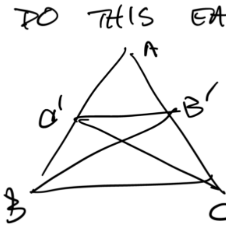
DIAGS $\overline{MB'}$ & $\overline{C'N}$ BISECT EACH OTHER

SO X = G.

TO GET ALL THREE CONCURRENT, DO IT AGAIN USING AB AS BASE. $\square$

SIMILARITIES TO HW7 #2.

CAN DO THIS EASIER USING SIMILARITY

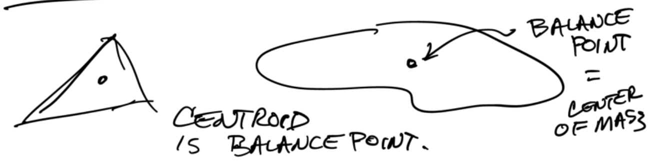


$\triangle GB'C' \sim \triangle GBC$

$$\frac{|BG|}{|B'G|} = \frac{|CG|}{|C'G|} = 2$$

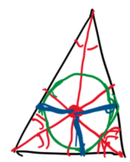
OFTEN YOU WANT TO TEACH THIS RESULT BEFORE YOU COVER SIMILARITY.

CENTROID: IN PHYSICS, CALCULUS



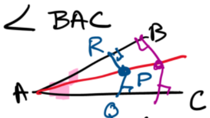
THM: THE THREE ANGLE BISECTORS ARE CONCURRENT AT THE INCENTER

THIS IS THE CENTER OF THE INSCRIBED CIRCLE



LEMMA: GIVEN $\angle BAC$

BISECTOR OF $\angle BAC$



$= \{P \mid \text{dist}(P, \overline{AB}) = \text{dist}(P, \overline{AC})\}$

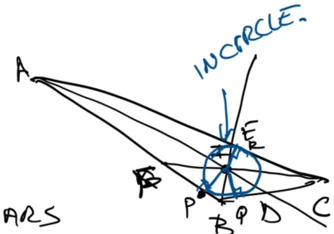
SINCE $\triangle PRA$ IS RIGHT, $\triangle PQA$ IS RIGHT

CONG BY AAS. $\llcorner \cong \llcorner$ $\angle A$ ANGLES $\cong$ AP IS SAME $\boxed{}$

$\Rightarrow |RP| = |PQ|$

LET BISECTORS OF $\angle B$ & $\angle C$ ($\overline{BE}$, $\overline{CF}$) MEET AT I

DROP PERPENDICULARS FROM I TO EACH SIDE.



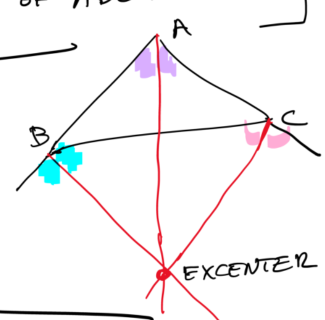
BUT $\underbrace{|IP| = |IQ|}_{\text{SINCE } I \in \text{BISECTOR OF } \angle B} \underbrace{= |IR|}_{\text{BISECTOR OF } C}$ $\boxed{}$

THERE ARE ALSO THREE EXCENTERS.

THM: IN $\triangle ABC$, LET l BE THE ANGLE BISECTOR OF $\angle A$, m & n BE THE BISECTORS OF EXTERIOR ANGLES AT B & C

THE l, m, n ARE CONCURRENT AT THE EXCENTER OF ABC REL A

Pf/ HW.
(AFTER EXAM)
SIM. TO PROOF FOR INCENTER.



THM: THE EULER LINE

LET H BE THE ORTHOCENTER
G BE THE CENTROID OF $\triangle ABC$
O BE THE CIRCUMCENTER

(i) IF $H = G = O$, THEN $\triangle ABC$ IS EQUILATERAL

(ii) IF $G \neq O$, THEN H LIES ON $\overline{OG}$ $H * G * O$

AND $|HG| = 2|GO|$