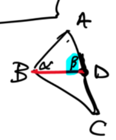
Monday, October 19, 2020 6:00 PM

THM: IN ANY TRIANGLE, THE ANGLE FACING THE LONGER SIDE HAS THE GREATER MEASURE, EG $|AC| > |AB| \iff \measuredangle B > \measuredangle C$



PF/ $\Rightarrow$ SPOSE $|AC| > |AB|$ WANT TO SHOW THAT $\measuredangle B > \measuredangle C$,

$\exists$ D ON $\overline{AC}$ SO THAT $|AD| = |AB|$

(COULD SHOW $D \in \angle ABC$ ...... BUT I WON'T)

LET $\measuredangle ABD = \alpha$ $\measuredangle ADB = \beta$

SINCE $|AB| = |AD|$, $\alpha = \beta$

$\angle ADB$ IS EXTERIOR TO $\triangle CDB$.

$\beta > \measuredangle C$

$\measuredangle B = \alpha = \beta > \measuredangle C$ $\boxtimes$

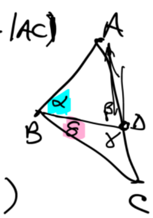
$\Leftarrow$ SPOZE $\measuredangle B > \measuredangle C$ BUT $|AC| \leq |AB|$ FOR CONTRADICTION.

IF $|AC| = |AB|$, THEN $\measuredangle B = \measuredangle C$ $\Rightarrow\Leftarrow$

IF $|AC| < |AB|$, THEN BY (a) $\measuredangle B < \measuredangle C$ $\Rightarrow\Leftarrow$ $\boxtimes$

THM THE SUM OF THE LENGTHS OF TWO SIDES OF A TRIANGLE IS LONGER THAN THE THIRD

GIVEN $\triangle ABC$ THEN $|AB| + |BC| > |AC|$



IF $|AB| \geq |AC|$, ALREADY DONE

IF $|AB| < |AC|$, FIND D ON AC SO THAT $|AD| = |AB|$

(AS BEFORE $D \in \angle ABC$, SO ADJACENT)

DENOTE ANGLES MEASURE BY $\alpha, \beta, \gamma, \delta$.

SINCE $|AB| = |AD| \Rightarrow \alpha = \beta$

$|AC| = |AD| + |DC|$

TO SHOW

$|AB| + |BC| > |AC| \iff |AB| + |BC| > |AD| + |DC|$

$\iff |AD| + |BC| > |AD| + |DC|$

$\iff |BC| > |DC|$

$\iff \alpha > \delta$

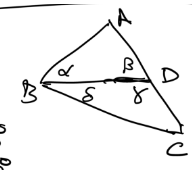
EXTERIOR TO ABD

$\gamma > \alpha = \beta > \delta$

EXTERIOR TO $\triangle BDC$

SO $\gamma > \delta$.

ANOTHER PROOF:



AS BEFORE $|AD| = |BC|$, $\alpha = \beta$

$\beta < 90°$ SINCE $\alpha + \beta + \measuredangle A = 180°$

$2\beta + \measuredangle A = 180°$

$2\beta < 180° \Rightarrow \beta < 90°$

BUT $\beta + \gamma = 180°$

SO γ IS OBTUSE.

SO $\delta < 90°$, IE $\gamma > \delta$.

THIRD PROOF THAT $\gamma > \delta$.

$\gamma + \beta = 180° \Rightarrow \gamma + \alpha = 180°$

BUT $\alpha + \delta = \measuredangle ABC$ SINCE ADJ.

$< 180°$

$\alpha + \delta < \gamma + \beta$, $\alpha = \beta$

SO $\delta < \gamma$

SO $|DC| < |BC|$ $\boxtimes$

THM: THE TRIANGE INEQUALITY

FOR ANY THE POINTS A, B, C IN THE PLANE

$\text{dist}(A,B) \leq \text{dist}(A,C) + \text{dist}(C,B)$

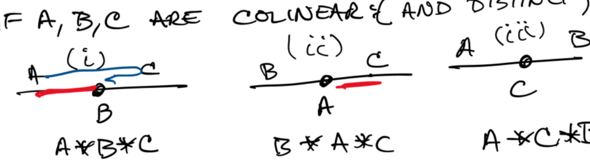
[$\text{dist}(A,B) = \text{dist}(A,C) + \text{dist}(C,B)$ $\iff$ A, B, C COLINEAR]

THM: FOR ANY A, B, C

$|AB| \leq |AC| + |CB|$ WITH EQUALITY $\iff A*C*B$.

PF/ IF A, B, C ARE NOT COLINEAR, JUST USE PREVIOUS ON $\triangle ABC$.

IF A, B, C ARE COLINEAR (AND DISTINCT)



IN (i) AND (ii) $|AB| < |AC| + |BC|$.

IN (iii) $|AB| = |AC| + |CB|$ $\leftarrow$

GOAL: SHOW THAT ISOMETRIES AND CONGRUENCES ARE THE SAME.

DEF: A TRANSFORMATION F HAS A FIXED POINT AT P IF $F(P) = P$. FOR SOME P.

EXAMPLES: IDENTITY $\iff$ EVERY POINT IS FIXED

CONST. $F(P) = C$ FOR ALL P $\iff$ C IS ONLY FIXED PT.

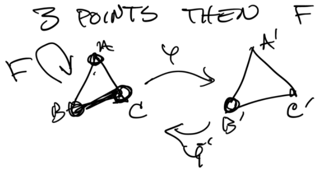
REFLECTION THRU ℓ $\iff$ ANY POINT ON ℓ IS FIXED

ROTATION ABOUT O BY $0 < \theta < 360$ $\implies$ O IS ONLY FIXED PT.

TRANSLATION A TO B $\iff$ NO FIXED POINTS.

WANT TO SHOW $\{\text{CONGRUENCES}\} = \{\text{ISOMETRIES}\}$

IDEA IF F IS ISOMETRY WHICH FIXES 3 POINTS THEN F IS THE IDENTITY



$F = \varphi^{-1} \circ \varphi = ID$

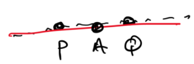
USE SSS TO SEE φ IS CONGRUENCE

TO PROVE: ANY ISOMETRY THAT FIXES 3 NON-COLINEAR POINTS, IS THE IDENTITY.

LEMMA

LET F BE AN ISOMETRY OF THE PLANE WHICH FIXES TWO DISTINCT POINTS P & Q THEN IF $A \in \overleftrightarrow{PQ}$, $F(A) = A$

CASE 1 $A \in \overline{PQ}$



LET $A' = F(A)$

WANT TO SHOW $A = A'$.

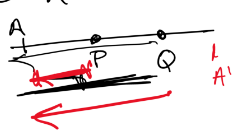
KNOW $|PA| + |AQ| = |PQ|$ SINCE $P * A * Q$.

(BUT $F(P) = P$, $F(Q) = Q$ AND $|F(\overline{XY})| = |XY|$

SO $|PA'| + |A'Q| = |PQ|$

$= |PA| + |A'Q|$ SO $A = A'$

CASE 2



$A * P * Q$

$A \in \overrightarrow{QP}$

$|PA| + |PQ| = |AQ|$ } F

$\implies |PA'| + |PQ| = |A'Q|$

AND $A' * P * Q$ (WHY? SINCE $|AQ| = |A'Q|$)

SO $A = A'$

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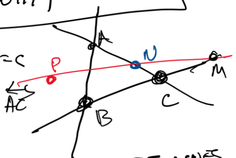
THM: IF F IS AN ISOMETRY WITH 3 NON-COLINEAR FIXED POINTS THEN F IS THE IDENTITY



ASSUME $F(A) = A$, $F(B) = B$, $F(C) = C$

FROM BEFORE ANY POINT ON $\overleftrightarrow{AB}$ OR $\overleftrightarrow{BC}$ OR $\overleftrightarrow{AC}$ IS FIXED

~~LET~~ LET P BE ANY POINT NOT ON THOSE LINES

WANT TO SHOW $F(P) = P$.

PICK SOME N ON $\overleftrightarrow{AC}$ ($N \neq A$, $N \neq C$)

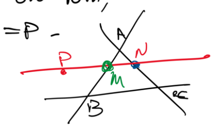
IF $\overleftrightarrow{NP} \cap \overleftrightarrow{BC}$ HAVE $M \in \overleftrightarrow{BC}$ AND $\overleftrightarrow{PN}$,

$F(N) = N$ SINCE IT FIXES ALL OF $\overleftrightarrow{AC}$

$F(M) = M$ - - - - - - $\overleftrightarrow{BC}$

SO F FIXES EVERY POINT ON $\overleftrightarrow{NM}$, IE $F(P) = P$.

WHAT IF $\overleftrightarrow{NP} \parallel \overleftrightarrow{BC}$?



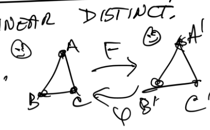
THEN USE $M = \overleftrightarrow{PN} \cap \overleftrightarrow{AB}$

AGAIN $F(M) = M$, $F(N) = N$, SO $F(P) = P$.

THM: EVERY ISOMETRY OF THE PLANE IS A CONGRUENCE

PF/ LET A, B, C BE NON-COLINEAR DISTINCT.

$F(A) = A'$, $F(B) = B'$, $F(C) = C'$.



SINCE F ISOM, $F(\overline{AB}) = \overline{A'B'}$ ETC.

$|F(AB)| = |A'B'|$ ETC... USE SSS

TO GET A CONGRUENCY $\varphi : \Delta A'B'C' \to \Delta ABC$

PROVIDED ABC WAS AND $A'B'C'$ ARE Δ

BUT BY Δ-INEQUALITY,

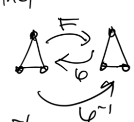
$|AB| + |BC| > |AC|$ AND $|A'B'| + |B'C'| > |A'C'|$

NOT COLINEAR

SO $\varphi \circ F = Id$

SINCE φ IS A CONGRUENCE

φ^{-1} IS ALSO.



$F = (\varphi^{-1} \circ \varphi) \circ F = \varphi^{-1} \circ \underbrace{(\varphi \circ F)}_{ID} = \varphi^{-1}$

SO F IS A CONGRUENCE.

IE ANY ISOMETRY OF PLANE IS JUST A COMPOSITION OF REFL, ROTATIONS, TRANSLATIONS. ■