Proof of FTS

Monday, October 12, 2020 10:11 AM

MIDTERM ON 28TH, IN CLASS.
- STATE SOME DEFS, THM.
- A PROB FROM HW
- ONE OR TWO OTHERS.

LAST TIME:

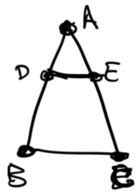
THM: THREE NON-COLINEAR POINTS UNIQUELY DEFINE A CIRCLE CONTAINING THEM

Pf/ DRAW TRIANGLE ΔABC.
THE PERPENDICULAR BISECTORS OF EACH SIDE ALL MEET AT THE CIRCUMCENTER.

THIS IS EQUIDISTANT FROM EACH VERTEX. ∎

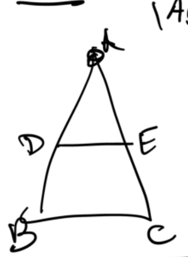
WANT TO PROVE F.T.S.

FTS: GIVEN A, B, C. LET D, E BE ON RAYS $\overrightarrow{AB}$ AND $\overrightarrow{AC}$, $D \neq B, A$ $E \neq C, A$
IF $\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|} = r$ $(r > 0)$
THEN $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$ AND $\frac{|DE|}{|BC|} = r$.



$r > 1$ or $r < 1$.
WE WILL PROVE FOR RATIONAL $r = \frac{m}{n}$.
TO GET $r \in \mathbb{R}$, NEED TO TAKE LIMIT.

NOTE: $\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|} \iff \frac{|AD|}{|AB|} - 1 = \frac{|AE|}{|AC|} - 1$
$\iff \frac{|AD|}{|AB| - |AD|} = \frac{|AE|}{|AC| - |AE|}$
$\iff \frac{|AD|}{|DB|} = \frac{|AE|}{|EC|}$

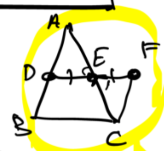


COR: A, B, C, D, E AS BEFORE
IF $\frac{|AD|}{|DB|} = \frac{|AE|}{|EC|}$ THEN $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

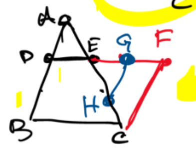
PROOF: FIRST PROVE $r = \frac{1}{k}$, $k \geq 2$.

LEMMA: LET $k \in \mathbb{Z}^+$, $k \geq 2$. SPOSE IN ΔABC, THERE IS D, E ON $\overline{AB}$, $\overline{AC}$ WITH $|AB| = k|AD|$ AND $|AC| = k|AE|$.
THEN $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$ AND $|BC| = k|DE|$

Pf/ WE ALREADY DID $k=2$.
TRICK WAS TO SHOW ▱DBCF IS A PARALLELOGRAM



USE THIS TO GET $k=3$, THIS GIVES THE IDEA OF THE INDUCTION.



EXTEND $\overline{DE}$ TO GET DF WITH $|DF| = 3|DE|$
WANT TO SHOW THAT ▱DBCF IS PARALLELOGR USING $k=2$.
LET $G = \text{MID}(E, F)$. SO $|DE| = |EG| = |GF|$
$H = \text{MID}(E, C)$ SO $|EH| = |HC| = |AE|$

NOW SHOW THAT $|BD| = |CF|$ AND $\overleftrightarrow{BD} \parallel \overleftrightarrow{CF}$

$\Delta ADE \cong \Delta HGE$

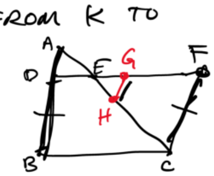
SINCE $\measuredangle AED = \measuredangle HEG$
$|DE| = |EG|$
$|AE| = |EH|$
} SAS — OR ROTATE 180° AROUND E

SO $|AD| = |GH|$
AND $\overleftrightarrow{AD} \parallel \overleftrightarrow{GH}$ (BY ROTATION OR ALT. INT. ANGLES)

APPLY $k=2$ TO  TO GET
$\overleftrightarrow{GH} \parallel \overleftrightarrow{FC}$ AND $2|GH| = |FC|$

BUT $|DB| = 2|AD|$ & $\overleftrightarrow{GH} \parallel \overleftrightarrow{AD} = \overleftrightarrow{AB}$
SO ▱DBCF IS A PARALLELOGRAM.

LETS DO IT AGAIN GOING FROM k TO $k+1$.
$\left(k|AD| = |DB|, \text{ i.e. } \frac{|AD|}{|AB|} = \frac{1}{k+1} \right)$



EXTEND $\overline{DE}$ TO $\overrightarrow{DF}$ WITH
$|EF| = k|DE|$

WANT TO SHOW THAT ▱DBCF IS A //OGRAM.
ADD G ON $\overline{DE}$ WITH $|DE| = |EG|$
H ON $\overline{EC}$ WITH $|AE| = |EH|$

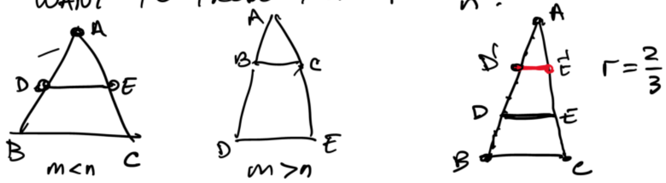
SO  HAS RATIO k. SO BY INDUCTION HYP, $\overleftrightarrow{GH} \parallel \overleftrightarrow{FC}$ AND $k|GH| = |FC|$.

ALSO, ROTATE 180° AROUND E (OR USE SAS, ALT. INT. ANGLES) TO GET $\Delta ADE \cong \Delta HGE$, $|AD| = |HG|$ AND $\overleftrightarrow{GH} \parallel \overleftrightarrow{AD}$

SO ▱DBCF IS A //OGRAM. ∎

SO WE KNOW FTS FOR $r = \frac{1}{n}$, $n = 2, 3, 4$

WANT TO PROVE FOR $r = \frac{m}{n}$.



SPOZE D, E ON $\overrightarrow{AB}$, $\overrightarrow{AC}$ $D \neq B$, $E \neq C$

MUST SHOW $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$ AND $\frac{|DE|}{|BC|} = m/n$

PICK D', E' ON $\overline{AB}$ AND $\overline{AC}$ WITH

$|AD'| = \frac{1}{n}|AB|$ $|AE'| = \frac{1}{n}|AC|$

KNOW $D'E' \parallel BC$ AND $|BC| = n|D'E'|$

NOW LOOK $\triangle ADE$

THE RATIO IS m:

$|AD| = m|AD'|$

$|AE| = m|AE'|$

$DE \parallel D'E'$

$\frac{|DE|}{|BC|} = \frac{m|D'E'|}{n|D'E'|} = \frac{m}{n}$. ∎

FOR $r \in \mathbb{R}$ NEED TO TAKE A LIMIT.

USE $\frac{1}{3}, \frac{10}{31}, \frac{100}{314}, \frac{1000}{3141}, \ldots$

TO DO THIS CAREFULLY

NEED IDEA OF LIMITS.

(SEE NOTES ON SCHEDULE)

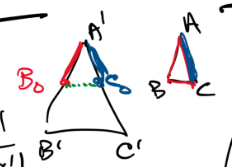
FROM III ~ $2\frac{1}{2}$ PAGES

CAN USE THIS TO GET

THM (SSS SIMILARITY)

IF $\triangle A'B'C' \sim \triangle ABC$

$\Longleftrightarrow \frac{|AB|}{|A'B'|} = \frac{|AC|}{|A'C'|} = \frac{|BC|}{|B'C'|}$



Pf. ASSUME $|A'B'| > |AB|$. (AND $\frac{|AB|}{|A'B'|} = \frac{|AC|}{|A'C'|} = \frac{|BC|}{|B'C'|}$)

ON $A'B'$, CONSTRUCT B_0 WITH $|A'B_0| = |AB|$

$A'C'$ CONSTRUCT C_0 WITH $|A'C_0| = |AC|$

NEED TO SHOW $|B_0C_0| = |BC|$

USE FTS:

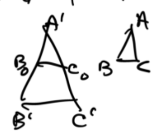
$\frac{|A'B_0|}{|A'B'|} = \frac{|A'C_0|}{|A'C'|} = r$ SO $\frac{|B_0C_0|}{|B'C'|} = r = \frac{|BC|}{|B'C'|}$

SO $|B_0C_0| = |BC|$

SO $\triangle ABC \cong \triangle A'B_0C_0$

BY SSS

SO $\angle A' = \angle A$



USE SAS SIMILARITY TO GET $\triangle A'B'C' \sim \triangle ABC$ ∎

FOR CONGRUENCE:

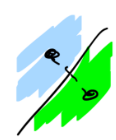
SSS, ASA, AAS

SIM. SSS AA

NEXT TOPIC IS TO SHOW

(SUM OF ANGLE MEASURE IN A TRIANGLE IS 180°.)

USES PARALLEL POSTULATE AND PLANE SEPARATION



FALSE IN HYPERBOLIC WHERE LOTS OF PARALLELS

FALSE IN SPHERICAL WHERE LINES DON'T SEPARATE, NO $\parallel$ LINES.