Medians,SSS,etc

Wednesday, October 7, 2020 12:28 PM

NOTE: MATERIAL TODAY AND FOLLOWING IS IN CH. 4.6 OF PUBLISHED TEXT, NOT ON A HANDOUT.

LAST TIME: A TRIANGLE IS ISOSCELES IF IT HAS TWO (OR MORE) CONGRUENT SIDES

IN $\triangle ABC$ IF $|AB| = |AC|$ THEN
A IS THE "TOP VERTEX"
$\overline{BC}$ IS THE BASE
$\measuredangle B$ & $\measuredangle C$ ARE THE BASE ANGLES

DEF: IN ANY TRIANGLE $\triangle ABC$

- THE (SEGMENT) LINE $\perp$ TO $\overleftrightarrow{BC}$ THRU A IS THE ALTITUDE (FROM A TO $\overline{BC}$)
- THE MEDIAN FROM A IS THE SEGMENT JOINING A TO THE MIDPOINT OF THE OPPOSITE SIDE
- PERPENDICULAR BISECTOR OF $\overleftrightarrow{BC}$
- BISECTOR OF $\measuredangle A$.

THM (G26) a) IN ANY ISOSCELES $\triangle$, THE BASE ANGLES ARE $\cong$

b) IN ANY ISOSCELES $\triangle$,

- THE PERP BISECTOR OF BASE ($\overleftrightarrow{MN}$)
- THE ANGLE BISECTOR OF THE TOP ($\overrightarrow{AG}$)
- THE MEDIAN OF THE BASE ($\overline{AM}$)
- THE ALTITUDE FROM THE BASE ($\overline{AD}$)

ARE ALL ON THE SAME LINE.

PF/(a) SPOZE $|AB| = |AC|$. (IE ISOSC)

LET $\overrightarrow{AG}$ BISECT ANGLE A, WITH G ON $\overline{BC}$ (THIS IS OK CUZ CROSSBAR)

USE SAS:

$|AB| = |AC|$ BY HYPOTHESIS
$\measuredangle BAG = \measuredangle CAG$ BY DEF.
$|AG| = |AG|$ SO BY SAS, $\triangle BAG \cong \triangle CAG$.
SO $\measuredangle B = \measuredangle C$

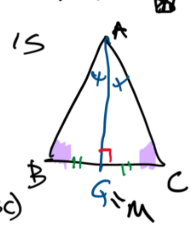
OR REFLECT THRU $\overrightarrow{AG}$ Λ

$\Lambda(\overrightarrow{AB}) = \overrightarrow{AC}$. SINCE ANGLES MATCH

$\Lambda(B) = B'$, SHOW $B' = C$ SINCE LENGTHS MATCH $\boxtimes$

(b) ALL THE OTHER STUFF IS THE SAME TOO.

IN PART (A) SHOWED $\triangle BAG \cong \triangle CAG$



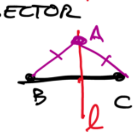
$\Rightarrow \overline{BG} \cong \overline{GC}$, SO $G = \text{MID}(BC)$

$\underbrace{\measuredangle BGA \cong \measuredangle CGA}_{\text{TOGETHER} = 180°}$ $\Rightarrow$ $\overleftrightarrow{AG}$ IS $\perp$ BISECTOR OF $\overline{BC}$

SO $\measuredangle G = 90°$

$\Rightarrow$ $\overline{AG}$ IS THE ALTITUDE AND THE MEDIAN. $\boxtimes$

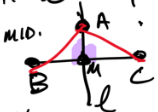
THM (G27) THE PERPENDICULAR BISECTOR OF A SEGMENT $\overline{BC}$ IS THE SET OF ALL POINTS THAT ARE EQUIDISTANT FROM B AND C



CAN THINK OF THE BISECTOR AS ALL POSSIBLE TOP VERTICES OF AN ISOSC. TRIANGLE WITH BASE $\overline{BC}$

PF/ LET ℓ BE THE $\perp$ BISECTOR OF $\overline{BC}$
LET $\mathcal{E}$ BE THE SET OF ALL POINTS
$\mathcal{E} = \{X \mid |BX| = |CX|\}$. SHOW THAT $\ell = \mathcal{E}$.

FIRST: $\ell \subset \mathcal{E}$. LET $A \in \ell$, $M = \text{MID}$.



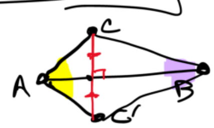
KNOW $\measuredangle BMA = \measuredangle CMA = 90°$
$|BM| = |CM|$, $|AM| = |AM|$
$\Rightarrow \triangle ABM \cong \triangle ACM$, SO $|AB| = |AC|$.

ALSO: $\mathcal{E} \subset \ell$.
TAKE $K \in \mathcal{E}$. SO $|BK| = |KC|$.
IF K IS ON $\overline{BC}$, THEN $K = M \in \ell$, SO DONE.
IF $K \neq M$, THINK ABOUT $\triangle BKC$.
IT IS ISOSCELES, WITH BASE $\overline{BC}$
SO MEDIAN $\overleftrightarrow{KM}$ = $\perp$ BISECTOR OF $\overline{BC}$

SINCE $\mathcal{E} \subset \ell$ AN $\ell \subset \mathcal{E}$, $\mathcal{E} = \ell$. $\boxtimes$

THM: (SSS) IN $\triangle ABC$ AND $\triangle A'B'C'$ IF THESE HAVE 3 PAIRS OF CONG. SIDES THEN $\triangle ABC \cong \triangle A'B'C'$

CASE 1: SPOZE $A = A'$
$B = B'$
C & C' IN OPP. HALF PLANES REL $\overleftrightarrow{AB}$



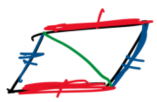
NOTE $\left.\begin{array}{l}|AC| = |AC'| \\ |BC| = |BC'|\end{array}\right\}$ SO $\overleftrightarrow{AB}$ IS $\perp$ BISECTOR OF CC'

SO $\overrightarrow{AB}$ IS THE BISECTOR OF ISOSC. $\triangle CC'B$ AND $\triangle CC'A$.
SO $\measuredangle CAB = \measuredangle C'AB$, $\measuredangle CBA = \measuredangle C'BA$
SO $\triangle$'S CONGRUENT BY ASA.

REST IS EASY.

- IF C, C' ARE IN SAME HALFPLANE AND $A = A'$, $B = B'$, JUST REFLECT TO GET INTO CASE ☆
- IF $A \neq A'$ AND/OR $B \neq B'$ USE A TRANSLATION AND/OR ROTATION TO MAKE IT SO AND BE IN ☆ ~~OR~~

COR: A QUADRILATERAL IS A PARALLELOGRAM $\iff$ BOTH PAIRS OF OPPOSITE SIDES ARE CONGRUENT



Pf/ PUT IN DIAGONAL, USE SSS.

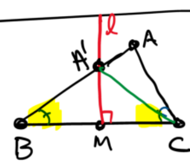
COR: A RHOMBUS IS A PARALLELOGRAM.

RHOMBUS = 4 EQUAL LENGTH SIDES.

(G29)

THM IF TWO ANGLES IN A TRIANGLE ARE CONGRUENT, THEN THE TRIANGLE IS ISOSCELES.

CONVERSE OF G26(a).



Pf/ LET M = MIDPOINT(BC)
$\ell = \perp$ BISECTOR$(\overline{BC})$

LET ℓ INTERSECT AB AT A'

IF $A = A'$ DONE, SINCE POINTS ON ℓ ARE EQUIDISTANT FROM B & C (SO $|AB| = |AC|$)

IF $A \neq A'$, WLOG ASSUME A AND C IN SAME HALFPLANE REL ℓ.

SINCE $A' \in \ell$, $|A'B| = |A'C|$, SO $\Delta A'BC$ IS ISOSC.

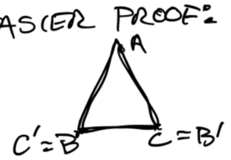
SINCE ISOSC., $\measuredangle B = \measuredangle BCA'$
$= \measuredangle BCA$ BY HYPOTHESIS.

SO $\overrightarrow{CA'} = \overrightarrow{CA}$

ALSO A, A' BOTH ON $\overleftrightarrow{AB}$, SO $A = A'$

WAY EASIER PROOF: (PAPPUS ~200 AD?)

SHOW $\Delta ABC \cong \Delta ACB$



$|BC| = |CB|$
$\measuredangle C' = \measuredangle B = \measuredangle C = \measuredangle B'$

USE ANGLE-SIDE-ANGLE TO SEE ΔABC IS CONG. TO ITS MIRROR.

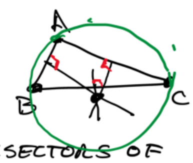
USELESS

~~DEF~~: A TRIANGLE WITH THREE EQUAL ANGLES IS CALLED EQUIANGULAR.

USEFUL: A TRIANGLE WITH 3 EQUAL SIDES IS EQUILATERAL.

THM: ALL EQUIANGULAR TRIANGLES ARE EQUILATERAL AND VICE VERSA

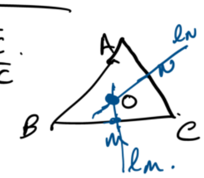
(HW)



THM.

(i) THE THREE PERPENDICULAR BISECTORS OF A TRIANGLE ALL MEET AT A POINT EQUIDISTANT FROM THE THREE VERTICES. THIS POINT IS THE CIRCUMCENTER

(ii) THERE IS EXACTLY ONE CIRCLE CONTAINING ALL THREE VERTICES OF A TRIANGLE. (CIRCUMSCRIBED CIRCLE) THE CIRCUMCIRCLE (CIRCUMCENTER IS ITS CENTER)



Pf/ (i) LET M = MIDPOINT OF $\overline{BC}$.
N = MIDPOINT OF $\overline{AC}$

$\ell_M = \perp$ BISECTOR AT M
$\ell_N = \perp$ BISECT. AT N
$\ell_M \cap \ell_N \ni O$.

(NOTE, MUST MEET COZ IF $/\!/$, THEN $\overleftrightarrow{AC} /\!/ \overleftrightarrow{BC}$)

NOW $|OB| = |OC|$ BECAUSE $\overleftrightarrow{OM}$ = EQUIDIST. FROM B & C

$|OA| = |OC|$ - - - $\overleftrightarrow{ON}$ = PERP. $\perp$

SO $|OA| = |OB| = |OC|$ ☒

(ii) IS EASY, NEXT TIME.