SAS,AA,PythagoreanThm

Monday, October 5, 2020 2:43 PM

LAST TIME: TWO SETS S AND S' ARE SIMILAR IF THERE IS A FINITE SEQUENCE OF CONGRUENCES & DILATION F SO THAT $F(S) = S'$

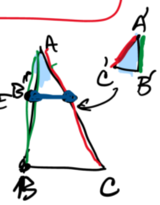
THM: TWO TRIANGLES ARE SIMILAR IF ALL CORRESPONDING ANGLE HAVE SAME MEASURE & CORR. SIDES ARE IN SAME PROPORTION

THIS IS FOR SURE TOO MUCH.

THM: (SAS SIMILARITY) $\Delta ABC \sim \Delta A'B'C'$ $\iff$ $\measuredangle A = \measuredangle A'$ AND $\dfrac{|AB|}{|A'B'|} = \dfrac{|AC|}{|A'C'|}$

PF/ $\Rightarrow$ FROM DEFINITION.

$\Leftarrow$ (IDEA IS TO "PICK UP" ONE TRIANGLE AND PLACE IT ONTO THE OTHER VIA CONGRUENCE (IE REFLECTION, TRANS, ROTATION) & SEE A DILATION DOES THE JOB.)



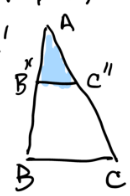
- IF $|AB| = |A'B'|$ THEN TRIANGLES ARE CONGRUENT SO DONE.

CAN ASSUME $|AB| > |A'B'|$.

FIND B'' ON $\overline{AB}$ SO THAT $|AB''| = |A'B'|$

C'' ON $\overline{AC}$ SO THAT $|AC''| = |A'C'|$

SO BY SAS, $\Delta AB''C'' \cong \Delta A'B'C'$



LET Δ BE THE DILATION WITH CENTER A, RATIO $r = \dfrac{|AB''|}{|AB|}$

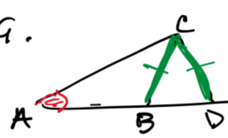
NOW $\Delta(A) = A$, $\Delta(B) = B''$, $\Delta(C) = C''$

SO $\Delta(\Delta ABC) = \Delta AB''C'' \underset{\varphi}{\cong} \Delta A'B'C'$

SO WE HAVE $\varphi(\Delta(ABC)) = \Delta A'B'C'$. ∎

NOTE THAT ANGLE HAS TO BE THE INCLUDED ANGLE. A THING.

ASS IS NOT



$\Delta ABC \ncong \Delta ADC$

BUT

$\measuredangle A = \measuredangle A$

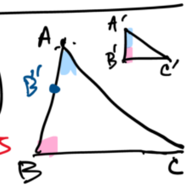
$|AC| = |AC|$

$|CB| = |CD|$.

THM: (AA SIMILARITY) TWO TRIANGLES WITH A TWO PAIRS OF CORR. CONGRUENT ANGLES ARE SIMILAR

(ONCE WE KNOW THAT THE SUM OF ANGLES IN A $\Delta = 180°$ CAN ALSO HAVE AAA SIM.)

SOMETIMES ITS CALLED THIS.



PF/ IF $|A'B'| = |AB|$, DONE BY ASA & CONGRUENT.

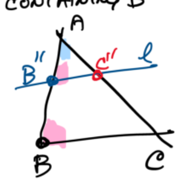
USE SAME TRICK: MOVE $\Delta A'B'C'$ ONTO ΔABC BY CONGRUENCE CHECK THAT A DILATION WORKS.

ASSUME $|AB| > |A'B'|$.

FIND B'' ON $|AB|$ SO THAT $|AB''| = |A'B'|$

FIND LINE ℓ PARALLEL TO $\overleftrightarrow{BC}$ CONTAINING B''

LET C'' BE THE INTERSECTION OF ℓ WITH $\overleftrightarrow{AC}$



$\measuredangle B = \measuredangle B''$ SINCE THEY ARE CORRESPONDING ANGLES

SO $\Delta AB''C'' \cong \Delta A'B'C'$ BY ASA.

NOW USE FTS* TO GET

$$\frac{|A'B'|}{|AB|} = \frac{|AB''|}{|AB|} = \frac{|AC''|}{|AC|}$$

THIS IS WHAT I NEED.

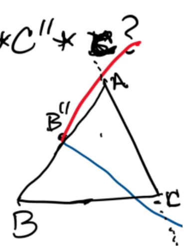
SO $\Delta ABC \sim \Delta AB''C'' \cong \Delta A'B'C'$. ∎

POSSIBLE ISSUE:

HOW DO WE KNOW THAT $A * C'' * C$?

PROVING THIS IS ANNOYING.

(DOING THIS IS EX 5 ON P. 376 OF NOTES. HAS LOTS OF HINTS)



ALL OF THIS STUFF (AA, ASA SIM) DEPENDS ON PARALLEL POSTULATE

(IN HYPERBOLIC GEOMETRY, SIMILARITY & CONGRUENCE ARE SAME)

SAS USES Δ TO MAP LINE TO A // LINE. (SO USES // POST.)

ASA
AA USES FTS* AGAIN DEPENDS ON // POSTULATE

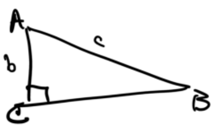
(ALSO, THERE SSS SIMILARITY... LATER)

LETS CHANGE THE TOPIC

PYTHAGOREAN THM

- THERE ARE ~~THOUSANDS~~ ~~HUNDREDS~~ LOTS OF DIFFERENT PROOFS OF THIS.
- BABYLONIANS USED IT IN c. 1900–1600 BC
- PYTHAGORAS LIVED c. 560–480 BC. (MAYBE? HE PROVIDED FIRST PROOF?)
- EUCLID'S PROOF (c. 300 BC) IS FIRST WRITTEN PROOF. IN ELEMENTS TWO DIFFERENT PROOFS OF THM.

TERMINOLOGY

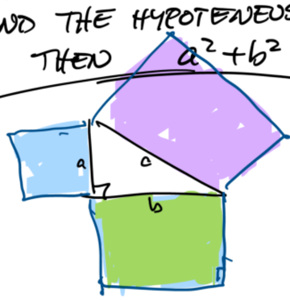


LET ΔABC BE A RIGHT TRIANGLE WITH $\measuredangle C = 90°$

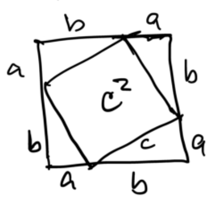
$|AB| = c$
$|AC| = b$
$|BC| = a$

$\overline{AC}$ AND $\overline{BC}$ ARE LEGS
$\overline{AB}$ IS HYPOTENEUSE

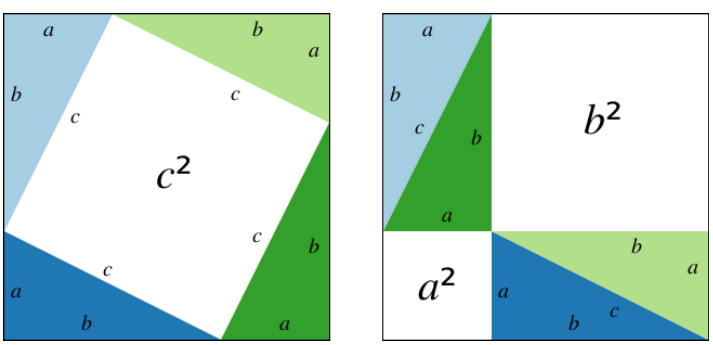
PYTHAGOREAN THEOREM: IF THE LENGTHS OF LEGS OF A RIGHT TRIANGLE ARE a & b, AND THE HYPOTENEUSE HAS LENGTH c THEN $a^2+b^2=c^2$



BLUE + GREEN AREA AREA
= PURPLE AREA.



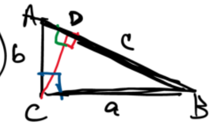
TROUBLE IS WE HAVEN'T TALKED ABOUT AREA.



For an animated version, see `https://www.math.stonybrook.edu/~scott/mat515.fall20/pythag.php`

LETS GIVE A PROOF USING SIMILARITY.

Pf/ GIVEN RT ΔABC ($\measuredangle C = \llcorner$)

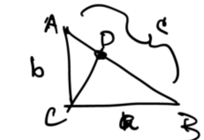


DRAW PERPENDICULAR TO $\overleftrightarrow{AB}$ THROUGH C, CALL INTERSECTION WITH $\overline{AB}$ D. (* MAYBE D IS NOT ON $\overline{AB}$ SEE BOOK p. 373–374)

CLAIM THAT $\Delta CBD \sim \Delta ABC$ ($\measuredangle B = \measuredangle B$, $\measuredangle C = \measuredangle D$ ◻)
$\Delta ACD \sim \Delta ABC$ ($\measuredangle A = \measuredangle A$, $\measuredangle C = \measuredangle D$ ◻)

SO $\dfrac{|AB|}{|BC|} = \dfrac{|BC|}{|BD|} \iff |BC|^2 = |AB| \cdot |BD|$
$a^2 = c \cdot |BD|$

AND $\dfrac{|AB|}{|AC|} = \dfrac{|AC|}{|AD|} \iff |AC|^2 = |AB| \cdot |AD|$
$b^2 = c \cdot |AD|$



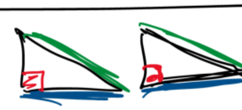
$a^2 + b^2 = c(|BD| + |AD|)$
$= c(|AB|) = c^2$ ⊠

THM: (CONVERSE OF PYTHAGOREAN THM)
IF ΔABC SATISFIES $|AB|^2 = |AC|^2 + |CB|^2$
THEN ΔABC IS $\measuredangle C = 90°$

Pf/ HW.

THM (HYPOTENEUSE-LEG HL)
IF ΔABC AND $\Delta A'B'C'$ ARE BOTH RIGHT TRIANGLES WITH THE HYPOTENEUSE OF EACH IS CONGRUENT AND AND A PAIR OF LEGS CONG., THEN $\Delta ABC \cong \Delta A'B'C'$

Pf/ (LOOKS LIKE ANGLE·SIDE·SIDE.)



BUT USE PYTHAG THM TO GET OTHER SIDES. APPLY SAS.

DEF: A TRIANGLE IS ISOSCELES IF IT HAS (TWO OR THREE) SIDES OF EQUAL LENGTH.

IF $|AB| = |AC|$, THEN $\overline{BC}$ IS CALLED ~~THE~~ BASE AND $\measuredangle B$ & $\angle C$ ARE THE BASE ANGLES (A IS THE TOP, $\angle A$ IS TOP ANGLE)