Dilations,Similarity

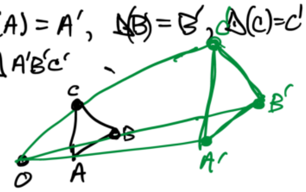
Wednesday, September 30, 2020 5:59 PM

LAST TIME:

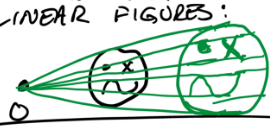
THM:
- DILATIONS MAP SEGMENTS TO SEGMENTS
- IF THE CENTER OF A DILATION $\Delta$ IS NOT ON $\overleftrightarrow{PQ}$ THEN $\Delta(\overleftrightarrow{PQ})$ IS PARALLEL TO $\overleftrightarrow{PQ}$

COR 1: ANY DILATION SENDS LINES $\Leftrightarrow$ LINES AND RAYS TO RAYS

COR 2: GIVEN $\triangle ABC$ & $\triangle A'B'C'$. IF $\Delta$ IS A DILATION SO THAT $\Delta(A) = A'$, $\Delta(B) = B'$, $\Delta(C) = C'$ THEN $\Delta(\triangle ABC) = \triangle A'B'C'$

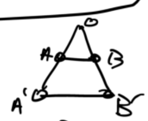



DILATIONS ARE NOT RECTILINEAR FIGURES:



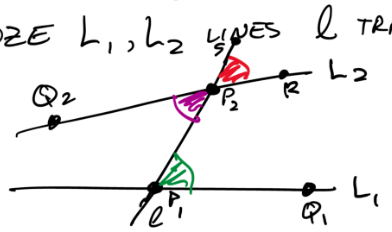

THM: LET $\Delta = \Delta_{O,r}$ CENTER O, SCALE r THEN
(i) $\Delta$ IS A BIJECTION OF THE PLANE. THE INVERSE IS THE DILATION W/ CENTER O SCALE $1/r$
(ii) FOR ANY $\overline{AB}$, WE HAVE $|\Delta(\overline{AB})| = r|AB|$
(iii) $\Delta$ SENDS ANGLES TO ANGLE AND PRESERVES ANGLE MEASURE

PF/ (i) JUST CHECK.
(ii) FOLLOWS FROM PREV.
(iii) NEED PROOF & SOME SETUP FIRST.




—— NAP ——

SPOZE $L_1, L_2$ LINES $\ell$ TRANSVERSE




$L_2 \cap \ell = P_2$
$L_1 \cap \ell = P_1$
$Q_1, Q_2$ ON $L_1, L_2$ OPPOSITE HALFPLANE OF $\ell$

DEF: $\angle Q_1P_1P_2$ AND $\angle Q_2P_2P_1$ ARE ALTERNATE INTERIOR ANGLES (OR OPPOSITE INTERIOR ANGLES)
$\angle Q_1P_1P_2$ AND $\angle RP_2S$ ARE CORRESPONDING ANGLES

CAN GIVE MORE PRECISE DEF TALKING ABOUT SAME HALF PLANE, ETC...

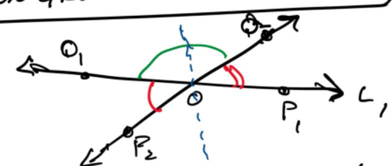
DEF: $\angle P_1P_2Q_2$ & $\angle RP_2S$ ARE CALLED VERTICAL ANGLES OR OPPOSITE ANGLES

THM: LET $L_1 \parallel L_2$ WITH $\ell$ BE TRANSVERSAL. THEN THE ALTERNATE INTERIOR ANGLES WRT $L_1, L_2, \ell$ ARE CONGRUENT (AS ARE CORRESPONDING ANGLE)

(FOR HW $\Leftarrow$)

THM: A PAIR OF VERTICAL ANGLES ARE CONGRUENT

PF/




IS A STRAIGHT ANGLE $(180°)$
$= 180°$
SO

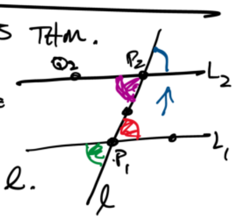
OR ROTATE BY $180°$ AROUND O
GOES TO

OR BISECT

& REFLECT

NOW PROVE ALT. INT ANGLES THM.

ASSUME $L_1 \parallel L_2$, $\ell$ TRANSVERSE
$P_1 = \ell \cap L_1$, $P_2 = \ell \cap L_2$
$Q_1$ & $Q_2$ IN OPP. HALF PLANES WRT $\ell$.




VICTORIA SAYS TRANSLATE RED TO BLUE.
USE TRANSLATION $T_{P_1P_2} = T$
$T(P_1) = P_2$ WANT T(RED) = BLUE
$T(L_1) = L_2$ BECAUSE THEY ARE PARALLEL AND $P_1 \in L_1$, $P_2 \in L_2$
$T(\ell) = \ell$ SO T(RED) = BLUE.
SO CORRESPONDING ANGLES ARE $\cong$.
BY VERTICAL ANGLES, RED $\cong$ PURPLE. ∎

ALTERNATIVELY ROTATE AROUND MIDPOINT OF $\overline{P_1P_2}$ BY $180°$

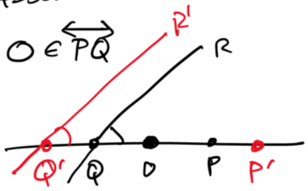
REMEMBER WE ARE TRYING TO PROVE

$\boxed{\text{ANY DILATION PRESERVES ANGLE MEASURE}}$

PF/ DILATIONS SEND RAYS TO RAYS, ANGLES → ANGLES FOR FREE.
GIVEN $\angle PQR$ AND $\angle P'Q'R'$ WITH
$\Delta(\angle PQR) = \angle P'Q'R'$
HAVE TO SHOW SAME MEASURE.

SEVERAL CASES.
ASSUME O IS NOT P, Q OR R.

IF $O \in \overleftrightarrow{PQ}$




$\Delta(P), \Delta Q$ ARE ON $\overleftrightarrow{PQ}$.
SO $\Delta(\overleftrightarrow{PQ}) = \overleftrightarrow{PQ}$
WHAT ABOUT $\overrightarrow{QR}$ VS $\overrightarrow{Q'R'}$?

$\angle Q'$ AND $\angle Q$ ARE CORRESPONDING ANGLES
SO $\measuredangle Q' = \measuredangle Q$.

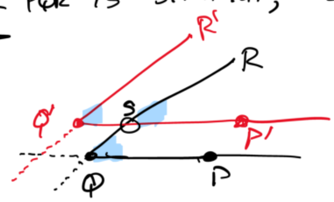
$\overrightarrow{QR} \parallel \overrightarrow{Q'R'}$

☺

SPOZE $\measuredangle PQR$ IS ZERO ANGLE, DONE
Q P R

$\Delta(\measuredangle PQR)$ = ZERO ANGLE.
$\measuredangle PQR$ IS FULL ANGLE, SAME
$\measuredangle PQR$ IS STRAIGHT, SAME.

GENERAL CASE




WE HAVE $\overleftrightarrow{PQ} \parallel \overleftrightarrow{P'Q'}$
AND $\overleftrightarrow{QR} \parallel \overleftrightarrow{Q'R'}$

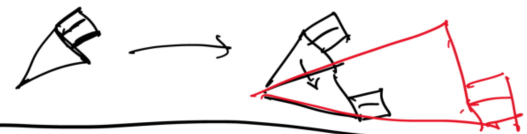
$\overleftrightarrow{QR} \cap \overleftrightarrow{Q'P'} = S$ ITS A TRANSVERSAL
(NOT PARALLEL CUZ THEN TO HAVE TWO LINES $\parallel$ TO $\overleftrightarrow{QP}$ CONTAINING Q')

SO $\angle PQR$ & $\angle P'SR$ ARE CORR. ANGLES
$\angle P'SR$ & $\angle P'Q'R'$ ARE CORR.
SO ALL $\cong$.

FOR HOMEWORK: CONVERSE TO ALT. INT. ANGLES THM

THM: IF THE ALTERNATE INTERIOR ANGLES OF A TRANSVERSAL WRT A PAIR OF LINES HAVE SAME MEASURE, THEN THE PAIR OF LINES ARE PARALLEL.
SAME IF CORRESPONDING ANGLES $\cong$

DEF: SETS S AND $S^*$ ARE SIMILAR ($S \sim S^*$) IF THERE IS A FINITE SEQUENCE OF DILATIONS AND CONGRUENCES F FOR WHICH $F(S) = S^*$



LEMMA: $\sim$ IS AN EQUIVALENCE RELATION.
THAT IS:
REFLEXIVE: FOR ANY A, $A \sim A$
SYMMETRIC: IF $A \sim B$, THEN $B \sim A$
TRANSITIVE: IF $A \sim B$ AND $B \sim C$, THEN $A \sim C$

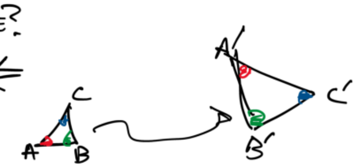
THM: FOR ANY TRANSFORMATION F THE FOLLOWING ARE EQUIVALENT.
(a) F IS A SIMILARITY
(b) F IS A DILATION FOLLOWED BY A CONGRUENCE
(c) F IS A CONGRUENCE FOLLOWED BY A DILATION.

$$\varphi \circ \Delta \circ \emptyset \circ \Delta' \circ \psi = \Delta'' \circ \varphi'$$
$$= \varphi' \circ \Delta'''$$

LONG TEDIOUS PROOF, NOT SURPRISING.

THM: GIVEN $\Delta ABC$ AND $\Delta A'B'C'$.
$\Delta ABC \sim \Delta A'B'C'$ IF AND ONLY IF
$\measuredangle A' = \measuredangle A$, $\measuredangle B' = \measuredangle B$, $\measuredangle C' = \measuredangle C$
AND $\dfrac{|AB|}{|A'B'|} = \dfrac{|AC|}{|A'C'|} = \dfrac{|BC|}{|B'C'|}$

HOW TO PROVE?
IDEA OF $\Leftarrow$




GIVEN ANGLES MATCH, SIDES IN RATIO, HOW TO SEE SIMILAR?