FTS,Dilations

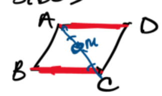
Monday, September 28, 2020 6:01 PM

LAST TIME, ENDED WITH

G13: A QUADRILATERAL IS A PARALLOGRAM $\iff$ ITS DIAGONALS BISECT EACH OTHER

NOW:

THM: A QUADRILATERAL IS A PARALLOGRAM $\iff$ A PAIR OF OPPOSITE SIDES ARE $\cong$ AND $\parallel$.



PF: $\Rightarrow$ ALREADY KNOW THIS (THM G4)

$\Leftarrow$ ASSUME $|AD| = |BC|$, $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

MUST SHOW THAT $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$

LET M BE THE MIDPOINT OF $\overline{AC}$

LET ρ BE THE 180° ROTATION AROUND M

· EASY TO SEE $\rho(A) = C$ ($\rho(\overleftrightarrow{AC}) = \overleftrightarrow{AC}$, $|AM| = |CM|$ ...)

WANT $\overleftrightarrow{BC}$ $\rho(\overleftrightarrow{AD}) \parallel \overleftrightarrow{AD}$ (G1)

AND $\rho(\overleftrightarrow{AD}) \ni C$, PARALLEL POSTULATE $\Rightarrow \rho(\overleftrightarrow{AD}) = \overleftrightarrow{BC}$

IE FLIP TOP, LINE IS THE BOTTOM

WANT $\rho(\overline{AD}) = \overline{BC}$

TO GET THAT $\rho(D) = B$, NEED TO KNOW $\rho(D)$ & B ARE IN SAME HALFPLANE OF $\overleftrightarrow{AC}$

TRUE CUZ D & $\rho(D)$ ARE IN OPPOSITE HALVES SO ARE D & B.

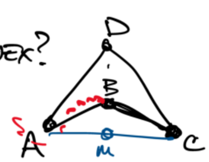
APPLY (G1), $\overleftrightarrow{BC} \parallel \overleftrightarrow{AB}$

SHIHA SAYS: DOES USING G13 WORK?

I THINK SO, BUT MORE WORK...

EVERYTHING OK?

WHAT IF $\square ABCD$ ISN'T CONVEX?



SO EVERYTHING ~~BREAKS~~ BREAKS CUZ B & D ARE SAME HALFPLANE.

PROBLEM IS DONT HAVE $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$

CUZ IF EXTEND $\overrightarrow{BC}$ IT INTERSECTS $\overline{AD}$?

CROSSBAR THM TELLS US:

IF $\overrightarrow{BC}$ IS INSIDE $\angle ACD$, THEN IT INTERSECTS $\overline{AD}$ SO NOT $\parallel$.

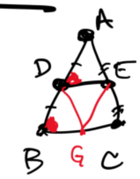
THM (FTS WITH $r = \frac{1}{2}$.)

LET $\triangle ABC$ HAVE $D = \mathrm{MID}(\overline{AB})$

$E = \mathrm{MID}(\overline{AC})$.

THEN $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$ AND

$2|DE| = |BC|$



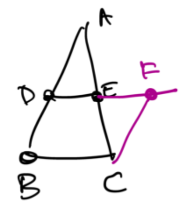
XUEER'S SUGGESTION: LET $G = \mathrm{MID}(BC)$ & SHOW ALL THOSE $\triangle$s ARE $\cong$

LETS TRY. $|AD| = |BD|$ HMM....

KATHERYN: THINK OF VECTORS $\langle AD\rangle + \langle AE\rangle = \langle DE\rangle$ ...

THIS CAN WORK, BUT AS LOTS OF TRANSLATIONS...

EXTEND $\overrightarrow{DE}$ TO F WITH $|DE| = |EF|$



NOW SHOW $\square DBCF$ IS A $\parallel$OGRAM.

IF SO $\overline{DF} \parallel \overline{BC}$, $|DE| = \frac{1}{2}|BC|$ SO GOOD

(XUEER: SHOW $\triangle ADE \cong \triangle CFE$ ...)

ROTATE $\overleftrightarrow{AD}$ AROUND E BY 180°

$\rho(A) = C$, $|AD| = |CF|$ .. ☺ ...

THM*: $\triangle ABC$ WITH $D = \mathrm{MID}(AB)$ AND ℓ PARALLEL TO $\overleftrightarrow{BC}$ THRU D INTERSECTS $\overleftrightarrow{AC}$ AT E

THEN $E = \mathrm{MID}(AC)$ AND $2|DE| = |BC|$

Pf/ LET T BE THE TRANSLATION WITH $T(A) = D$.

· SINCE $|AD| = |DB|$, $T(D) = B$.

ALSO $T(\overleftrightarrow{DE}) \parallel \overleftrightarrow{DE}$, SINCE $B \in T(\overleftrightarrow{DE})$

BY $\parallel$ POSTULATE $T(\overleftrightarrow{DE}) = \overleftrightarrow{BC}$

LET $T(E) = F$.

$T(\overleftrightarrow{AC}) \parallel \overleftrightarrow{AC} \Rightarrow T(\overline{AE}) = \overline{DF}$

SO DFCE IS A $\parallel$OGRAM

SO $|FC| = |DE| = |BF|$

$2|DE| = |BC|$.

NEW TRANSFORMATION, NOT ISOMETRY.

DEF A DILATION IS A TRANSFORMATION OF THE PLANE Δ, WITH A CENTER O, SCALE FACTOR r $(r>0)$.

(i) $\Delta(O)=O$ [O IS FIXED BY Δ]

(ii) IF $P \neq O$, AND $\Delta(P)=P'$

$$|OP'| = r|OP|$$

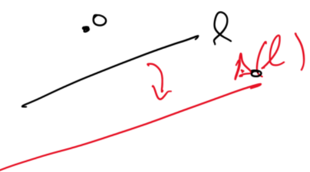
$r=1 \Rightarrow \Delta = \text{IDENTITY}$

~~$r=0 \Rightarrow \Delta = \text{CONSTANT}$ $\Delta_{90}(P)=O$~~ NO!

THM: DILATIONS MAP SEGMENTS TO SEGMENTS

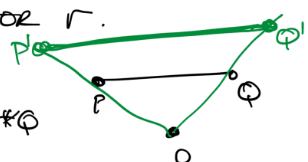
SPECIFICALLY, IF Δ IS A DILATION, IT SENDS $\overline{PQ}$ TO SEGMENT JOINING ΔP TO ΔQ.

ALSO IF $\overleftrightarrow{PQ}$ DOESN'T CONTAIN THE CENTER OF Δ, THEN $\Delta(\overleftrightarrow{PQ}) \parallel \overleftrightarrow{PQ}$



Pf

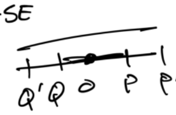
LET Δ = DILATION W/ CENTER O SCALE FACTOR r.



IF $O \in \overleftrightarrow{PQ}$, EITHER $O*P*Q$ OR $P*O*Q$

BUT IN EITHER CASE

$\Delta(\overline{PQ}) = \overline{P'Q'}$



ASSUME $O \notin \overleftrightarrow{PQ}$ AS IN

$\boxed{\text{NEED TO SHOW THAT } \Delta(\overline{PQ}) = \overline{P'Q'}}$

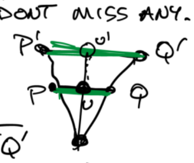
$\Delta(\overline{PQ}) \subset \overline{P'Q'}$ AND $\overline{P'Q'} \subset \Delta(\overline{PQ})$

NO EXTRA POINTS — DON'T MISS ANY.

TO SEE $\Delta(\overline{PQ}) \subset (\overline{P'Q'})$

TAKE ANY POINT $U \in \overline{PQ}$

AND SEE THAT $\Delta(U) = U' \in \overline{P'Q'}$



NOTE $\dfrac{|OP'|}{|OP|} = \dfrac{|OU'|}{|OU|} = r$

BY **FTS** $\overleftrightarrow{PU} \parallel \overleftrightarrow{P'U'}$ ⟷ SAME $\overleftrightarrow{PQ} \parallel \overleftrightarrow{P'U'}$

BUT $\overleftrightarrow{PQ} = \overleftrightarrow{PU}$

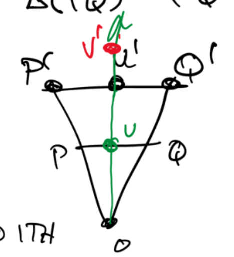
BY CROSSBAR, $\overrightarrow{OU} \cap \overline{P'Q'}$ AT SOME U^*

BUT $U^* = U'$ BY $\parallel$ POSTULATE.

SO $U' \in \overline{P'Q'}$, & $\Delta(\overline{PQ}) \subset \overline{P'Q'}$.

NOW TO SHOW

$\overline{P'Q'} \subset \Delta(\overline{PQ})$



TAKE $U' \in \overline{P'Q'}$

NEED TO SHOW THAT THERE IS $U \in \overline{PQ}$ WITH $\Delta(U) = U'$

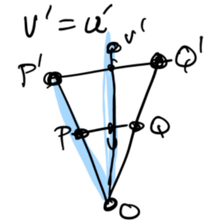
SO GET CANDIDATE U ON $\overline{PQ}$ BY INTERSECTION OF $\overrightarrow{OU'}$ WITH $\overline{PQ}$

MUST SHOW THAT $\dfrac{|OU'|}{|OU|} = r$.

LET V' BE A POINT $\overrightarrow{OU}$ WITH $|OV'| = r|OU|$

(JUST WHATEVER WORKS)

GOAL TO SHOW THAT $V' = U'$



IN $\triangle OP'V'$

APPLY FTS.

$\left|\dfrac{OP'}{OP}\right| = \dfrac{|OU'|}{|OU|} = r$

SO $\overleftrightarrow{PV'} \parallel \overleftrightarrow{PQ}$ BUT $\overleftrightarrow{P'V'} \ni P'$ AND $\overleftrightarrow{P'Q'} \parallel \overleftrightarrow{PQ}$

SO $\overleftrightarrow{PV'} = \overleftrightarrow{P'Q'}$ BY PARALLEL POSTULATE

SO $V' = U'$